Testování hypotéz

- H_0 – nulová hypotéza

$f(x|H_0)$

- H_1, H_2 .... – alternativní hypotézy

$f(x|H_1), f(x|H_2), \ldots$

- testovací statistika $t(x)$

chyba 1. druhu

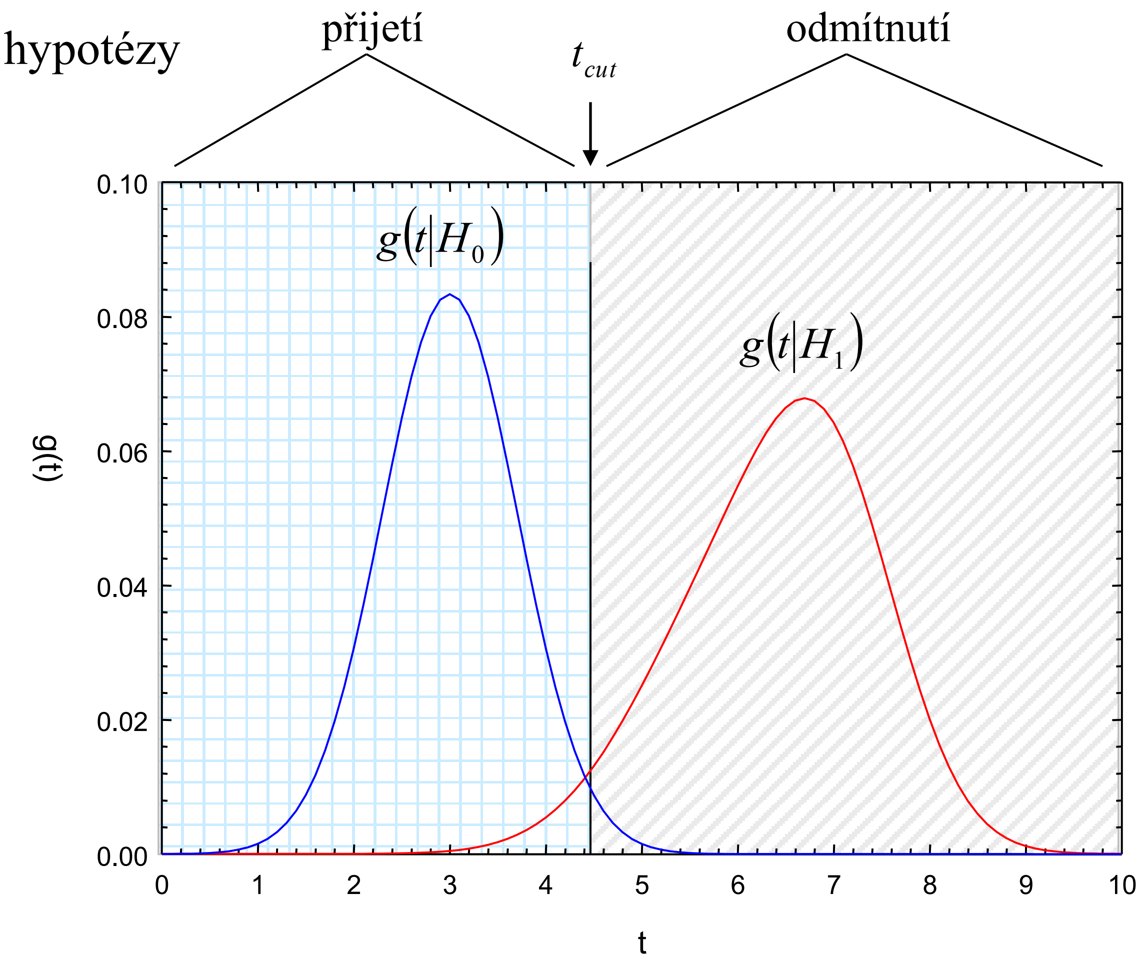
$$\alpha = \int_{t_{cut}}^{\infty} g(t|H_0)\,dt$$

signifikance

chyba 2. druhu

$$\beta = \int_{-\infty}^{t_{cut}} g(t|H_1)\,dt$$

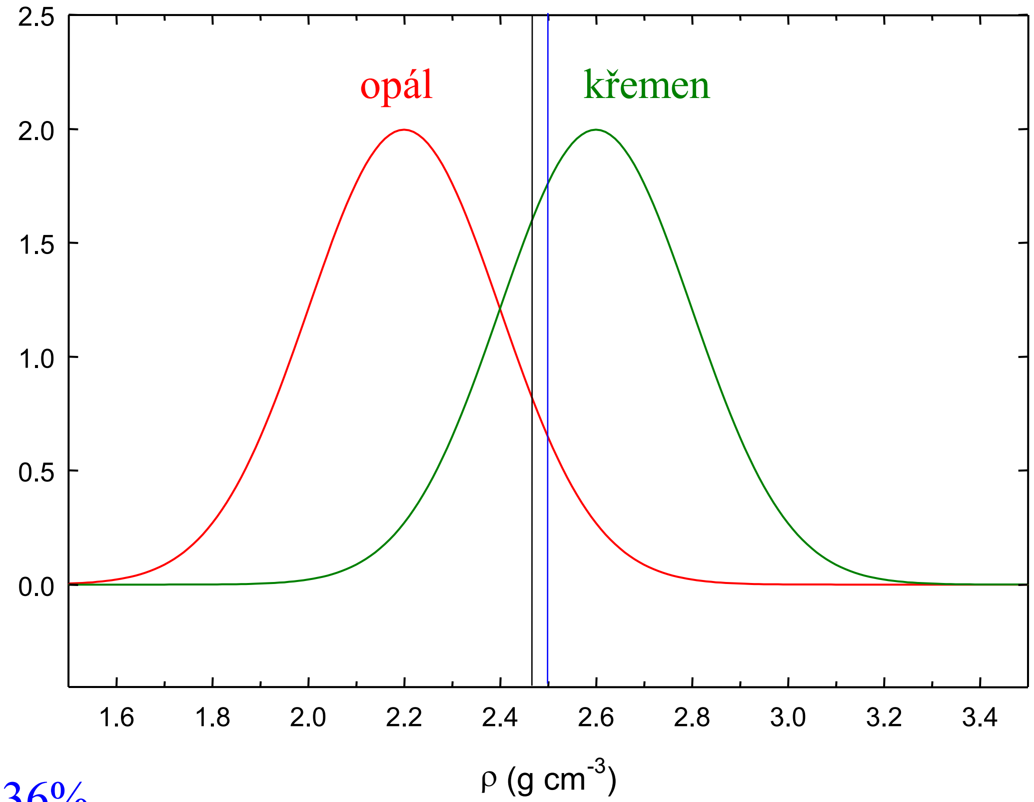
1-β : síla testu



Testování hypotéz

křemen vs. opál

- opál: $\rho = 2.2$ g cm^{-3}
- křemen: $\rho = 2.6$ g cm^{-3}
- chyba měření hustoty: 0.2 g cm^{-3}

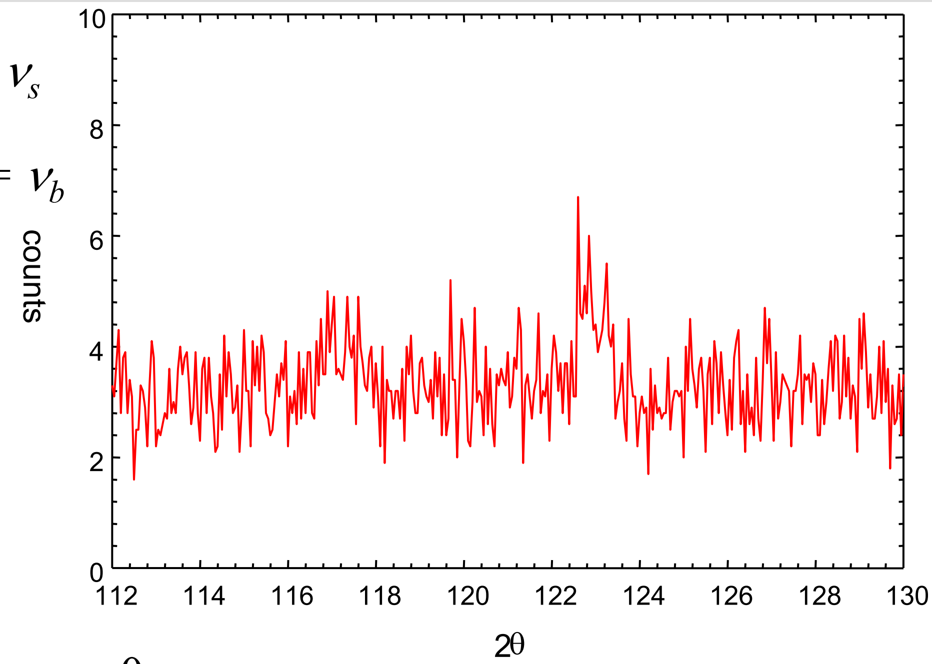


1. opál: $\rho \leq 2.50$ g cm^{-3} $\rightarrow$ $\alpha = 5\%$ $\beta = 36\%$

2. opál: $\rho \leq 2.45$ g cm^{-3} $\rightarrow$ $\alpha = 10\%$ $\beta = 24\%$

Nový efekt ???

- signál: n_s, Poissonovo rozdělení, $E[n_s] = \nu_s$
- pozadí: n_b, Poissonovo rozdělení, $E[n_b] = \nu_b$



$$n = n_s + n_b$$

$$E[n] = \nu_s + \nu_b$$

$$f(n|\nu_s, \nu_b) = \frac{(\nu_s + \nu_b)^n}{n!} e^{-(\nu_s + \nu_b)}$$

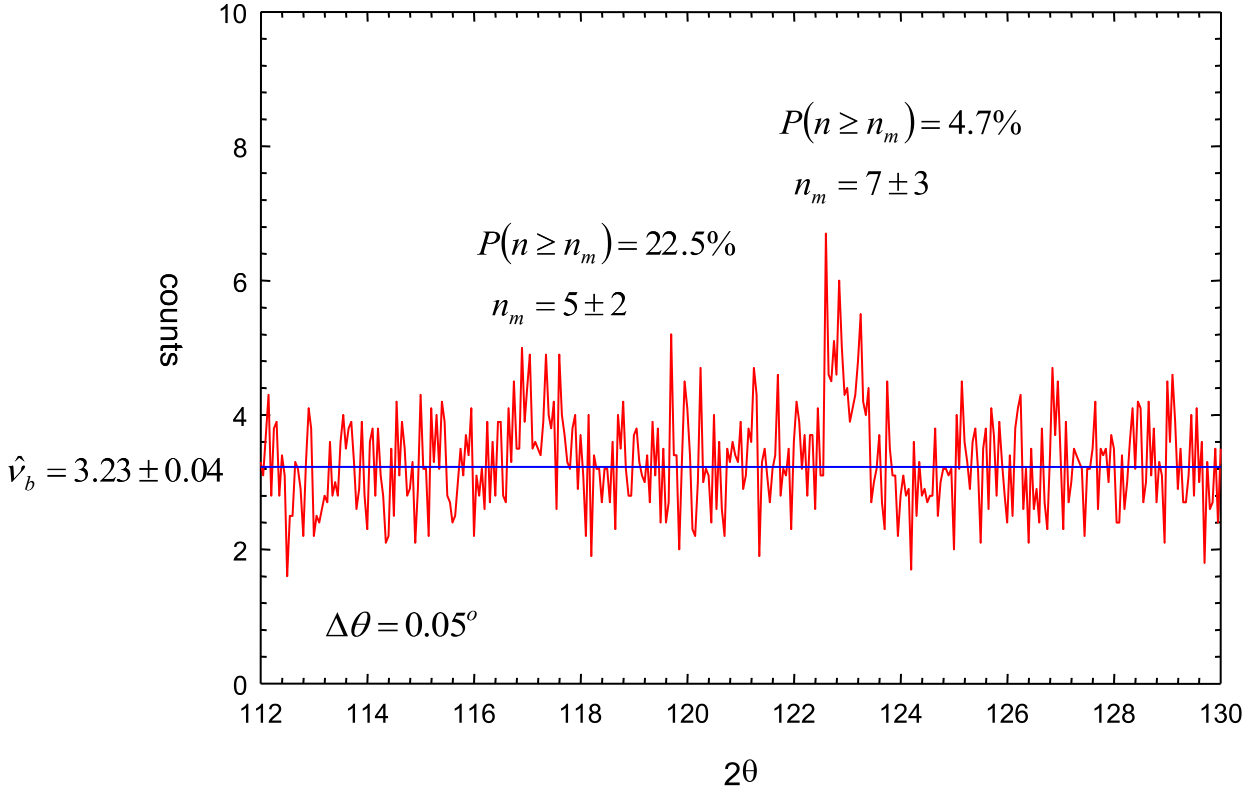
- nulová hypotéza: Není tam žádný efekt $\Rightarrow$ $\nu_s = 0$

$$P(n \geq n_m) = \sum_{n=n_m}^{\infty} f(n|\nu_s = 0, \nu_b) = 1 - \sum_{n=0}^{n_m - 1} f(n|\nu_s = 0, \nu_b) = 1 - \sum_{n=0}^{n_m - 1} \frac{\nu_b^n}{n!} e^{-\nu_b}$$

- např. $\nu_b = 0.5$ $n_m = 5 \Rightarrow P = 1.7 \times 10^{-4}$

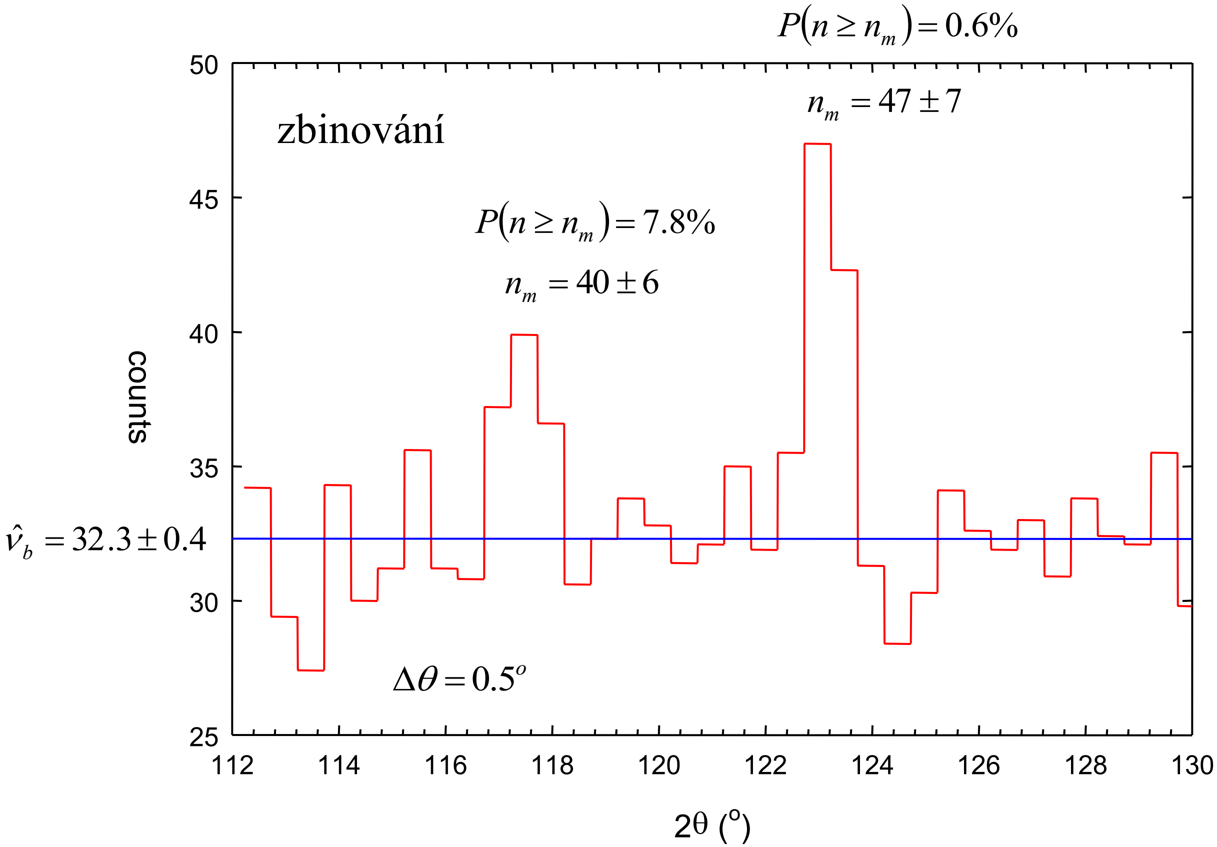
Nový efekt ???

$$P(n \geq n_m) = 1 - \sum_{n=0}^{n_m - 1} \frac{\nu_b^n}{n!} e^{-\nu_b}$$



Nový efekt ???

$$P(n \geq n_m) = 1 - \sum_{n=0}^{n_m - 1} \frac{\nu_b^n}{n!} e^{-\nu_b}$$



Normální rozdělení: Jsou dvě čísla stejná ?

$T_1 = (202 \pm 3)\ ^{o}C$

$T_2 = (209 \pm 4)\ ^{o}C$

$$\sigma^2_{\Delta T} = 9 + 16 = 25$$

$\Delta T = (7 \pm 5)\ ^{o}C$

$\Delta T = 1.4\ \sigma$

$P(|\Delta T|) \geq 1.4\ \sigma = 16\ \%$

Normální rozdělení: sada naměřených hodnot

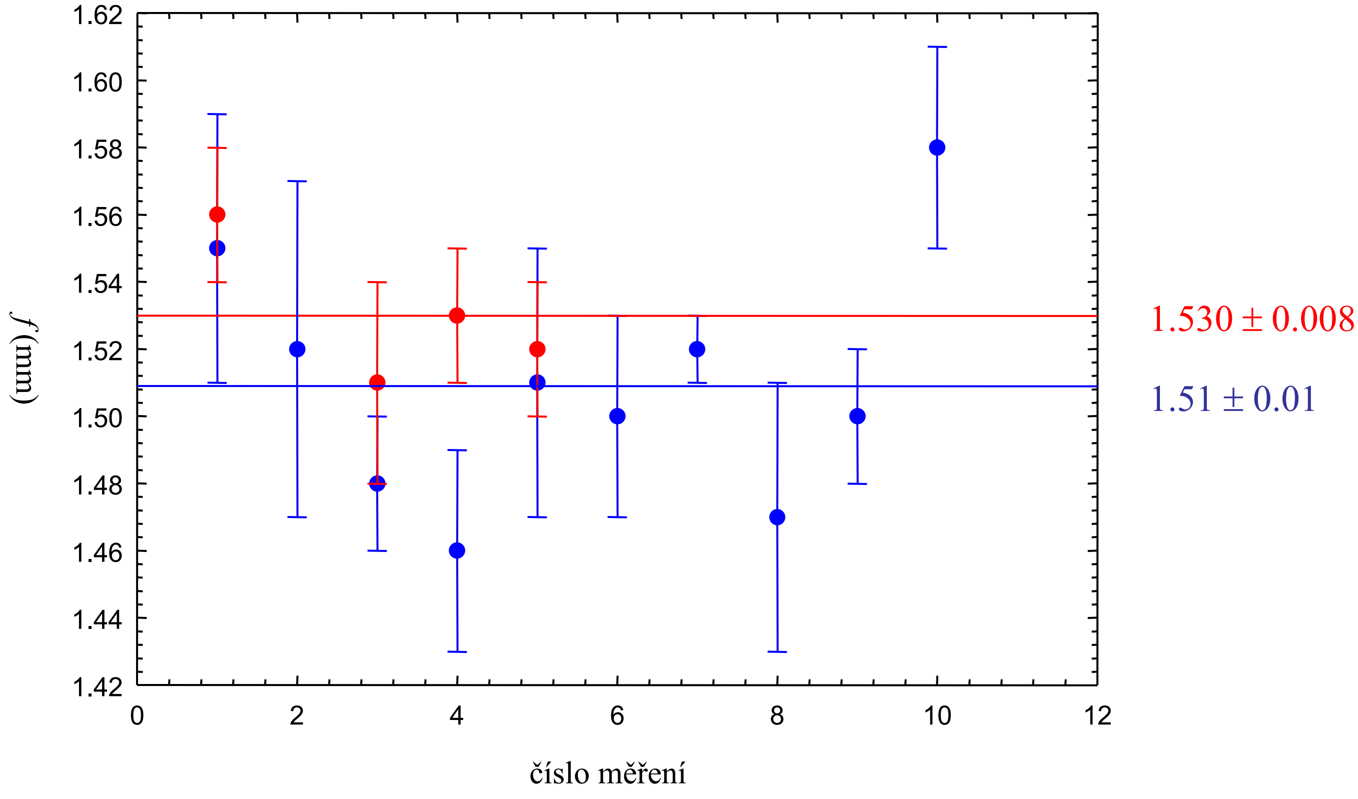
ohnisková vzdálenost f

$$\bar{f} = \frac{\sum_{i=1}^{N} \frac{f_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}$$

$$\sigma_{\bar{f}}^2 = \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}$$

$\Delta f = 0.02 \pm 0.01$

$\Delta f = 2\,\sigma$



Normální rozdělení: sada naměřených hodnot

$$H_0: \quad \mu_1 = \mu_2$$

$$m = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2 / N_1 + \sigma_2^2 / N_2}} \in N(0,1)$$

$$d = \frac{(N_1 - 1)s_1^2}{\sigma_1^2} + \frac{(N_2 - 1)s_2^2}{\sigma_2^2} \in \chi^2(N_1 + N_2 - 2)$$

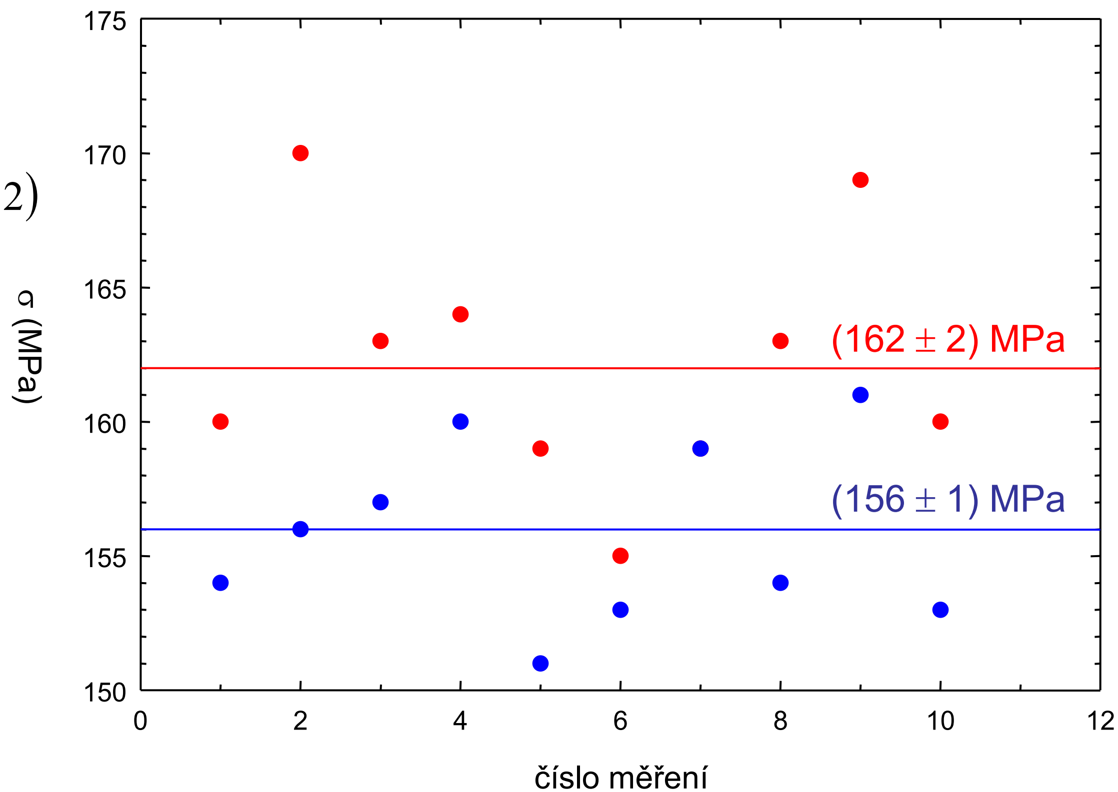
$$t = \frac{m\sqrt{N_1 + N_2 - 2}}{\sqrt{d}}$$

t : výběr ze studentova rozdělení

$$s^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N - 1}$$

slitina 1

slitina 2



Studentovo *t* rozdělení

$$t = \frac{x\sqrt{\nu}}{\sqrt{y}}$$

$$x \in N(0,1)$$

$$y \in \chi^2(\nu)$$

studentovo rozdělení s ν stupni volnosti

$$f(t|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}\left(1+\frac{t^2}{\nu}\right)^{-(\nu+1)/2}$$

- gama funkce

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma(x-1) = x\Gamma(x)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

Studentovo *t* rozdělení

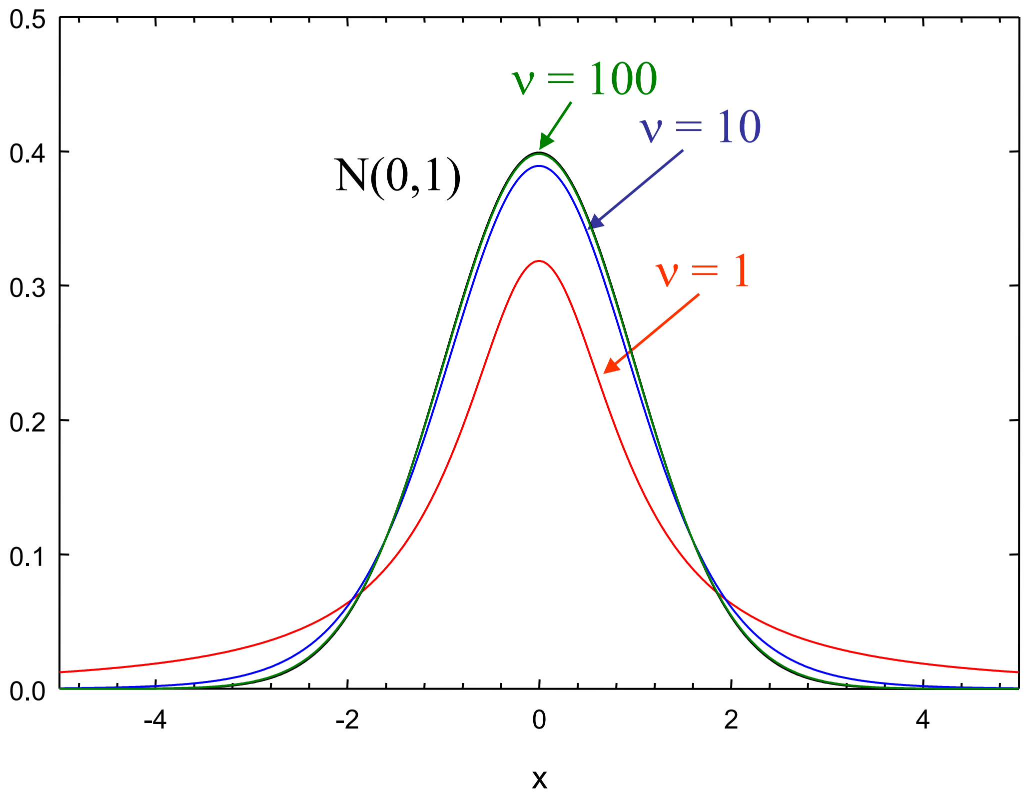
$$t = \frac{x\sqrt{\nu}}{\sqrt{y}}$$

$$x \in N(0,1)$$

$$y \in \chi^2(\nu)$$

studentovo rozdělení s ν stupni volnosti

$$f(t|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}\left(1+\frac{t^2}{\nu}\right)^{-(\nu+1)/2}$$



$$f(t|\nu=1) = \frac{1}{\pi}\frac{1}{1+t^2}$$

$$\nu \to \infty \quad f(t|\nu) \to N(0,1)$$

$$E[t] = 0$$

$$V[t] = \frac{\nu}{\nu-2} \qquad \nu > 2$$

Normální rozdělení: sada naměřených hodnot

$$H_0: \quad \mu_1 = \mu_2$$

$$m = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2 / N_1 + \sigma_2^2 / N_2}} \in N(0,1)$$

$$d = \frac{(N_1 - 1)s_1^2}{\sigma_1^2} + \frac{(N_2 - 1)s_2^2}{\sigma_2^2} \in \chi^2(N_1 + N_2 - 1)$$

$$t = \frac{m\sqrt{N_1 + N_2 - 2}}{\sqrt{d}}$$

t : výběr ze studentova rozdělení

$$\sigma_1 = \sigma_2 \equiv \sigma$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{1/N_1 + 1/N_2}}$$

$$S^2 = \frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}$$

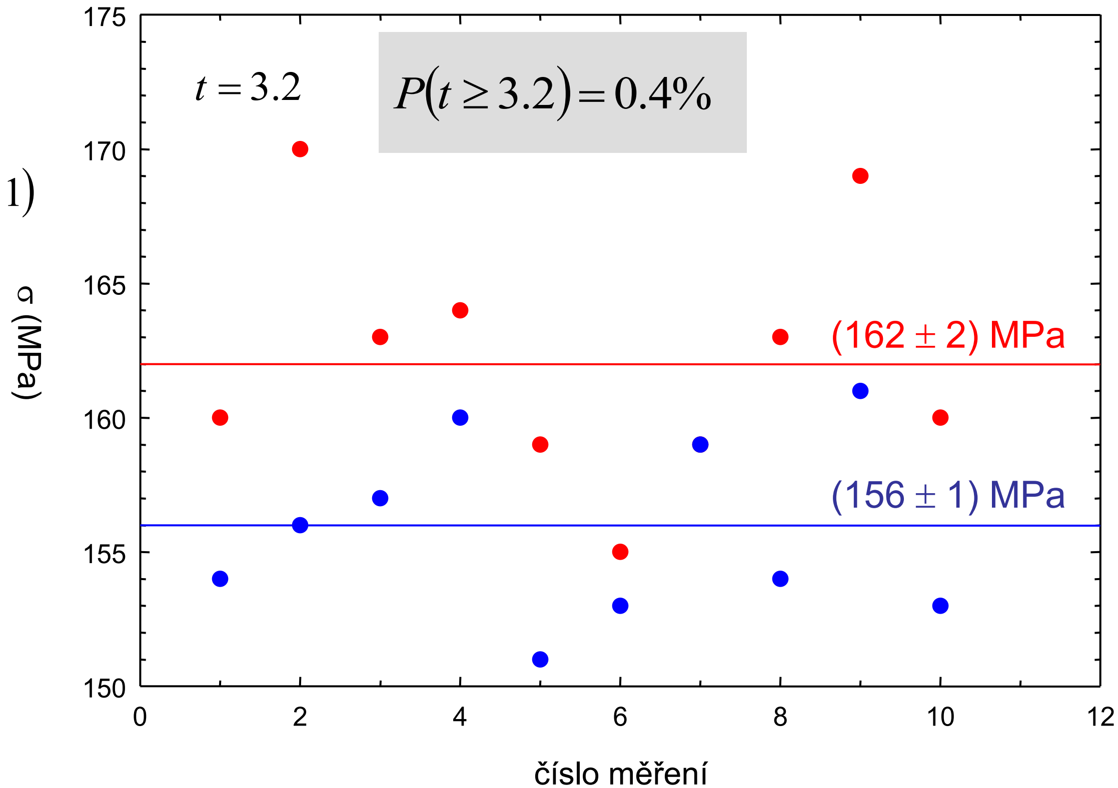
$$s^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}$$

$s_1 = 3.4$

$s_1 = 4.7$

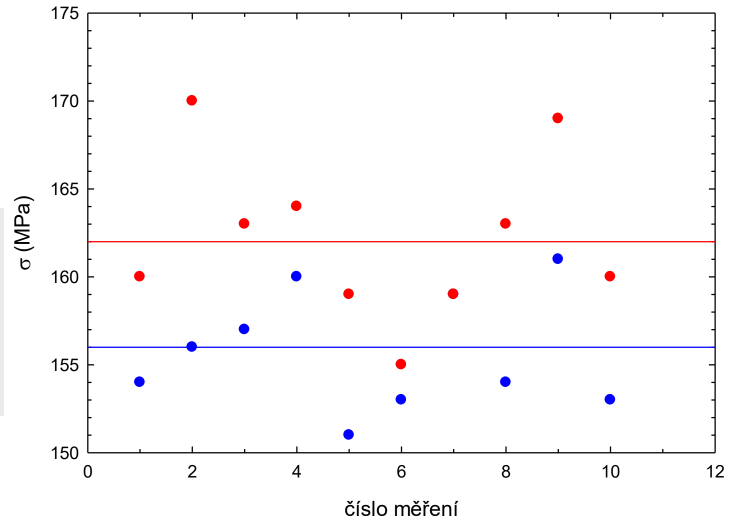
slitina 1

slitina 2



Fisherovo F rozdělení

• jsou rozptyly dvou sad naměřených hodnot stejné?



$$s^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}$$

$$F = 1.9$$

$$P(F \geq 1.9) = 23\%$$

$$F = \frac{s_1^2}{s_2^2}$$

$$f(F|N_1, N_2) = \frac{\Gamma\left(\frac{N_1+N_2}{2}\right)}{\Gamma\left(\frac{N_1}{2}\right)\Gamma\left(\frac{N_2}{2}\right)} \sqrt{N_1^{N_1} N_2^{N_2}} \frac{F^{\frac{N_1}{2}-1}}{(FN_1 + N_2)^{\frac{N_1+N_2}{2}}}$$

• velká N $\Rightarrow$ $\quad Z \equiv \frac{1}{2} \log F \quad Z \in N(\mu, \sigma) \quad \mu = \frac{1}{2}\left(\frac{1}{N_1} - \frac{1}{N_2}\right)$

$$\sigma^2 = \frac{1}{2}\left(\frac{1}{N_1} + \frac{1}{N_2}\right)$$

χ^2 test kvality fitu

- $x_1, x_2, \ldots x_N$ závislé proměnné
- $y_1, y_2, \ldots y_N$ naměřené hodnoty

$$y_i = N(\mu_i, \sigma_i)$$

- parametry: $\theta_1, \theta_2, \ldots \theta_m$
- modelová funkce: $\lambda(x|\mathbf{\theta})$
- testovací statistika: $\chi^2 = \sum_{i=1}^{N} \frac{\left(y_i - \lambda(x_i|\mathbf{\theta})\right)^2}{\sigma_i^2}$

$$\chi^2 \in \chi^2(N-m)$$

Rozdělení χ^2

$$f(y|n) = \frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} y^{\frac{n}{2}-1} e^{-\frac{y}{2}}, \quad y \in \langle 0,\infty), n = 1,2,\ldots$$

- gama funkce

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$$

$$\Gamma(n) = (n-1)!$$

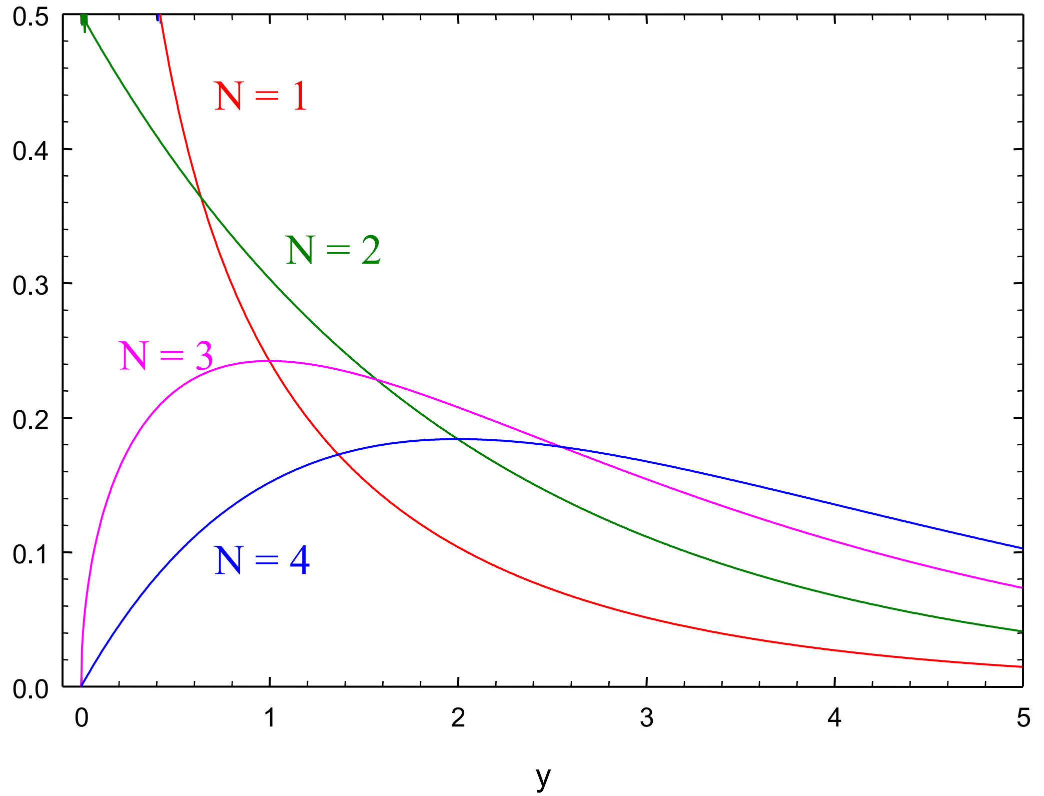
$$\Gamma(x-1) = x\Gamma(x)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

- n – počet stupňů volnosti

$$E[y] = n$$

$$V[y] = 2n$$



χ^2 test kvality fitu

- $x_1, x_2, \ldots x_N$ závislé proměnné
- $y_1, y_2, \ldots y_N$ naměřené hodnoty

$$y_i = N(\mu_i, \sigma_i)$$

- parametry: $\theta_1, \theta_2, \ldots \theta_m$
- modelová funkce: $\lambda(x|\boldsymbol{\theta})$

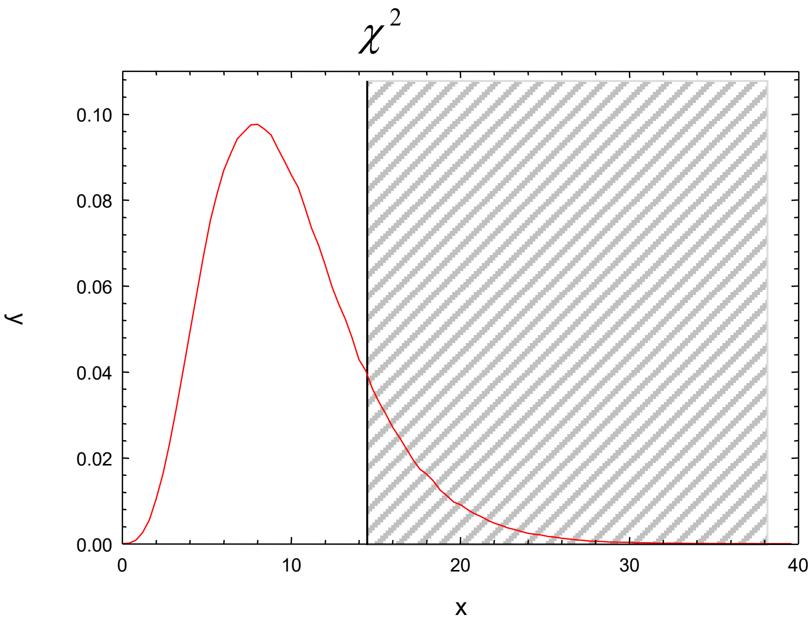
testovací statistika:

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - \lambda(x_i|\boldsymbol{\theta}))^2}{\sigma_i^2}$$

$$\chi^2 \in \chi^2(N-m)$$

$$E[\chi^2] = N - m$$

$$V[\chi^2] = 2(N-m)$$



$$P = \int_{\chi^2}^{\infty} f(z|N-m)\,dz$$

χ^2 na počet stupňů volnosti $\chi^2/(N-m)$

χ^2 test kvality fitu

$m = 2,\ \chi^2 = 47.04$

$\chi^2 / (N\text{-}m) = 5.88$

$P = 5.0 \times 10^{-6}$

$m = 3,\ \chi^2 = 36.47$

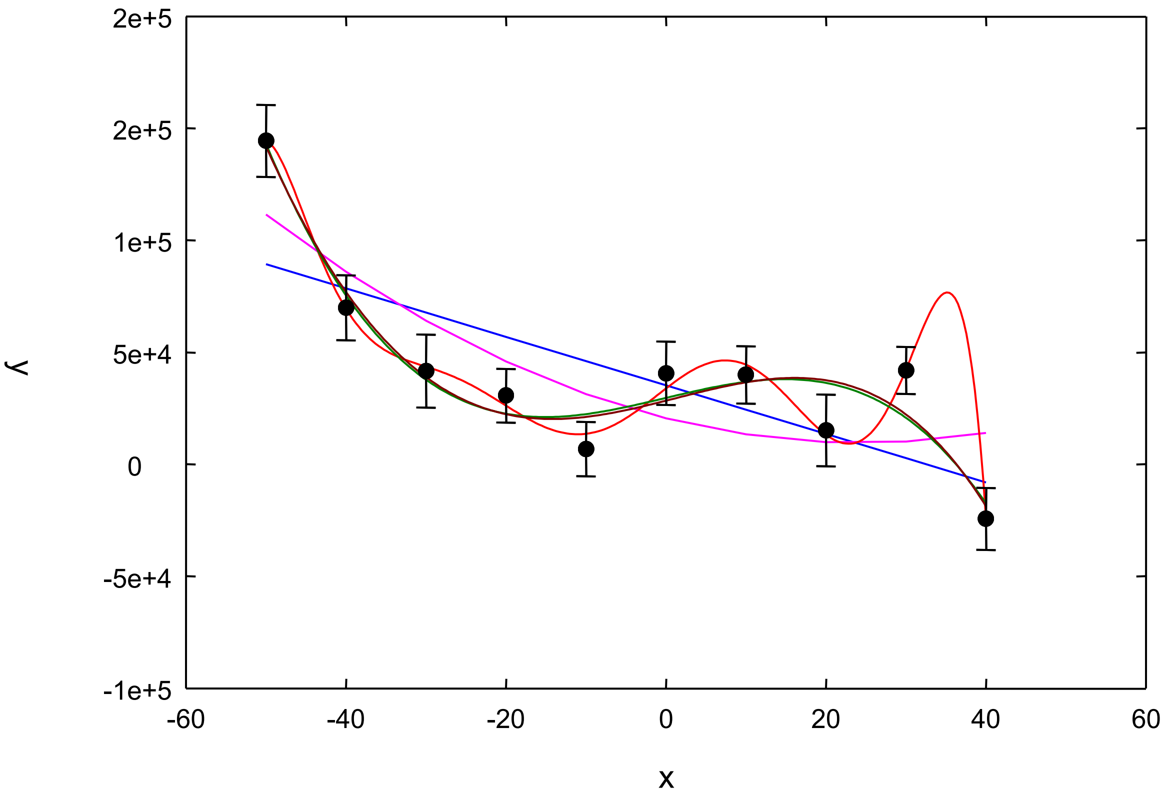
$\chi^2 / (N\text{-}m) = 5.21$

$P = 1.4 \times 10^{-5}$

$m = 4,\ \chi^2 = 9.06$

$\chi^2 / (N\text{-}m) = 1.51$

$P = 0.68$



$m = 5,\ \chi^2 = 8.60$

$\chi^2 / (N\text{-}m) = 1.72$

$P = 0.64$

$m = 9,\ \chi^2 = 7.56$

$\chi^2 / (N\text{-}m) = 0.84$

$P = 0.53$

χ^2 test kvality fitu – binovaná data

$$\chi^2 = \frac{\sum_{i=1}^{N} \left(y_i - f(\theta_i)\right)^2}{f(\theta_i)}$$

$$N = 201$$

$$\nu = 201 - 5 = 196$$

$$\chi^2(\nu)$$

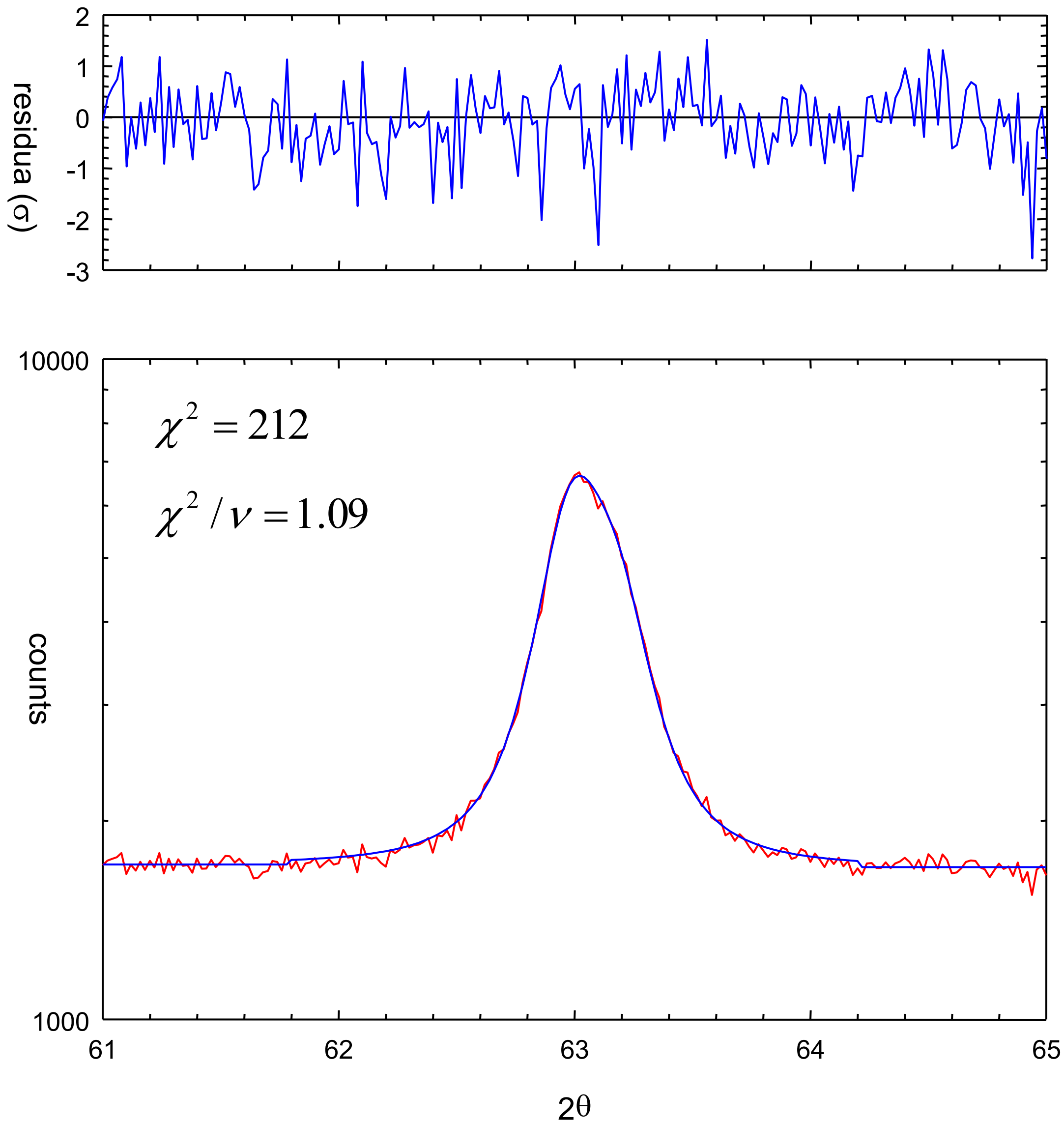
$$\mu = \nu = 196$$

$$\sigma = \sqrt{2\nu} = 20$$

$$\chi^2(\nu)/\nu$$

$$E\left[\chi^2(\nu)/\nu\right] = 1$$

$$\sigma_{\chi^2(\nu)/\nu} = 0.1$$



Kolmogorův test

• index lomu skla $N = 1.5192$

• odhadnutá chyba měření 6×10^{-4}

$$D = \max\left|\mathrm{cum}(x) - F(x)\right|$$

• 7 hodnot

$$D = 0.20$$

$$P(D \geq 0.20) = 91\%$$

$$N \geq 4$$

$$P(D \geq D_{obs}) = Q_K\left(\left(\sqrt{N} + 0.12 + 0.11/\sqrt{N}\right)D_{obs}\right)$$

$$Q_K(x) = 2\sum_{i=1}^{\infty}(-1)^{i-1} e^{-2i^2x^2}$$