Funkce náhodné proměnné

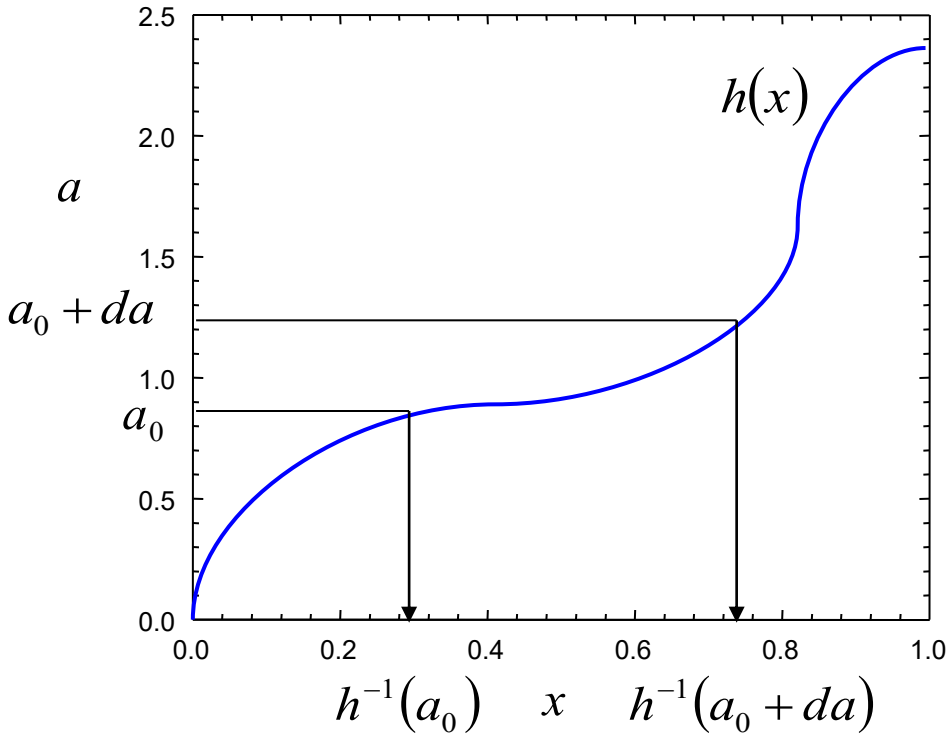
- stará náhodná proměnná: x
- hustota pravděpodobosti: $f(x)$
- nová náhodná proměnná: a

$$a = h(x)$$

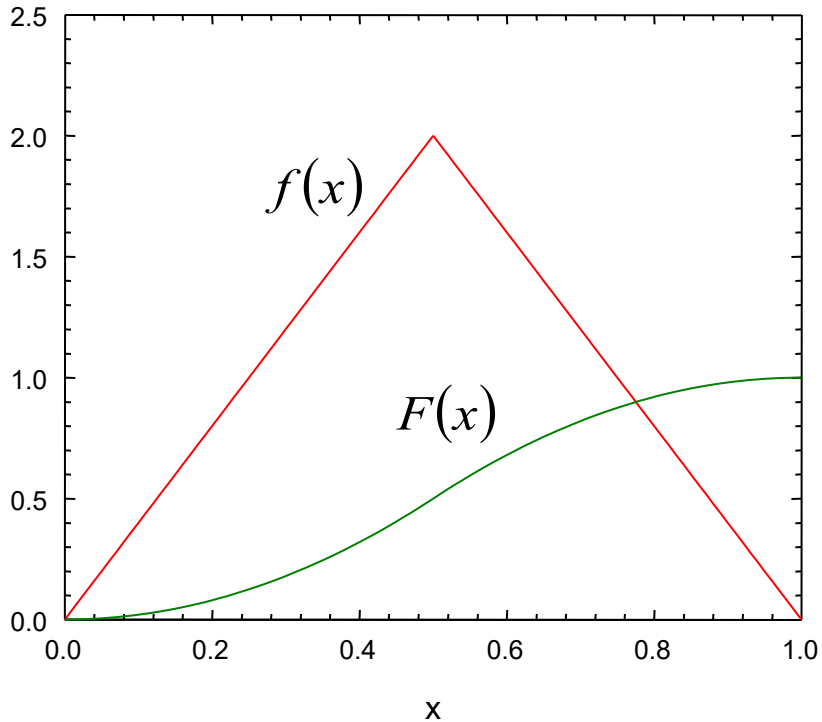
- hustota pravděpodobosti: $g(a)$

$$x = h^{-1}(a)$$

$$g(a) = f\left(h^{-1}(a)\right)\left|\frac{dh^{-1}}{da}\right|$$



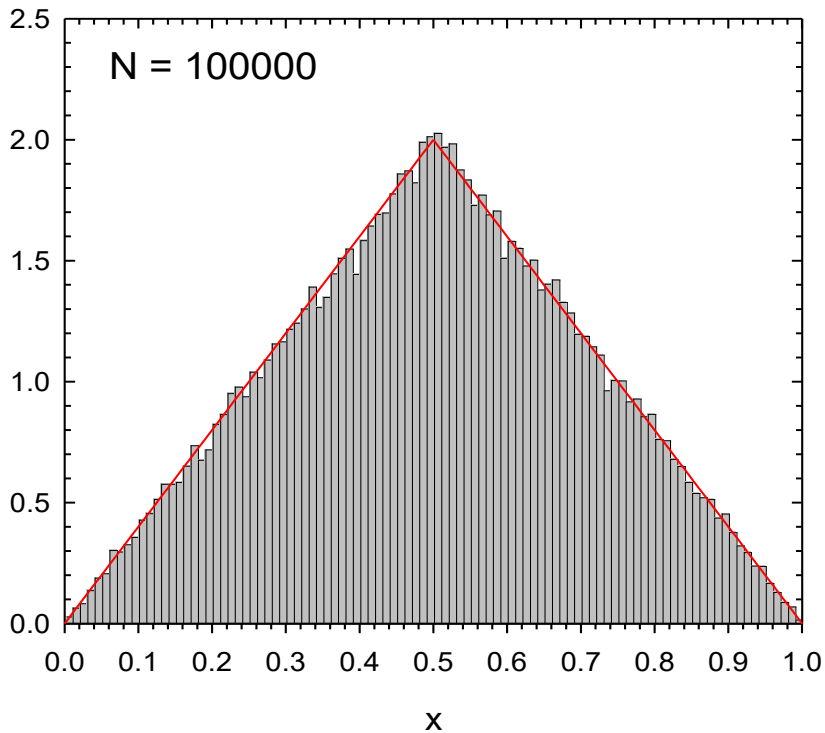
Funkce náhodné proměnné



$$f(x) = \begin{cases} 4x & 0 \le x \le \frac{1}{2} \\ -4x+4 & \frac{1}{2} < x < 1 \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

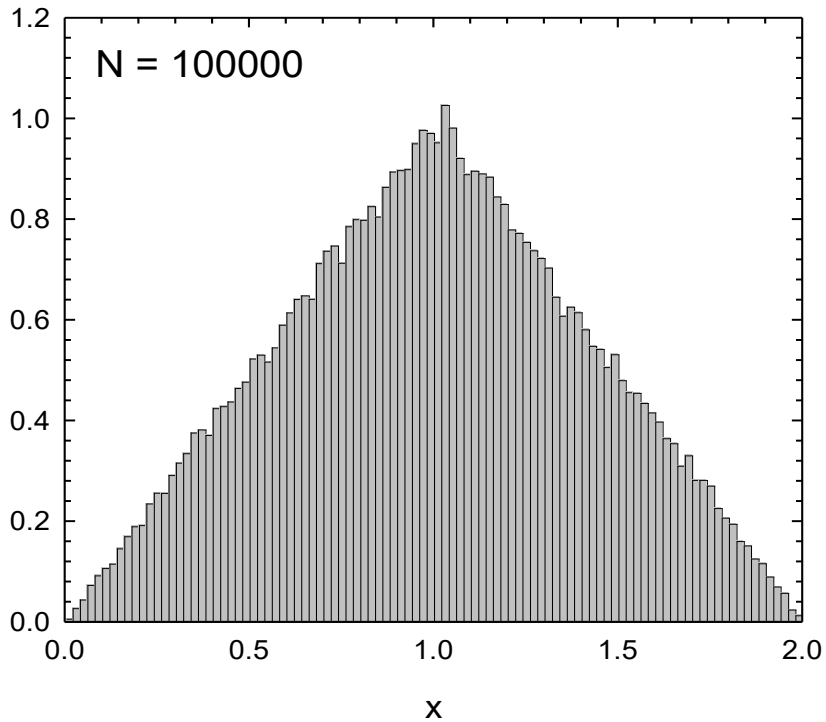
N = 100000
$\Delta = 0.01$



$$f(x) = \begin{cases} 4x & 0 \le x \le \frac{1}{2} \\ -4x+4 & \frac{1}{2} < x < 1 \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

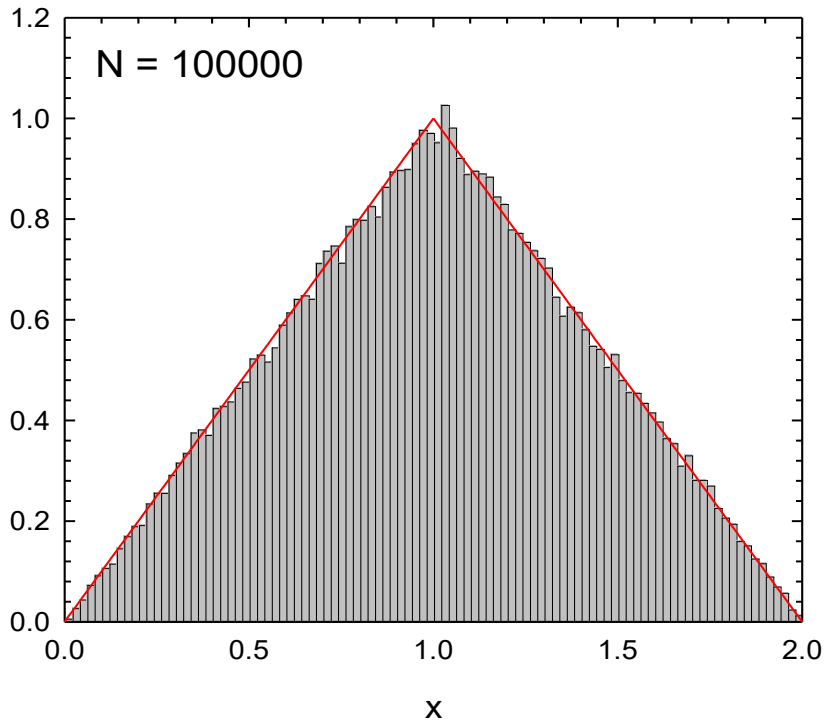
N = 100000
$\Delta = 0.02$



$$a = 2x$$

Funkce náhodné proměnné

N = 100000
$\Delta = 0.02$



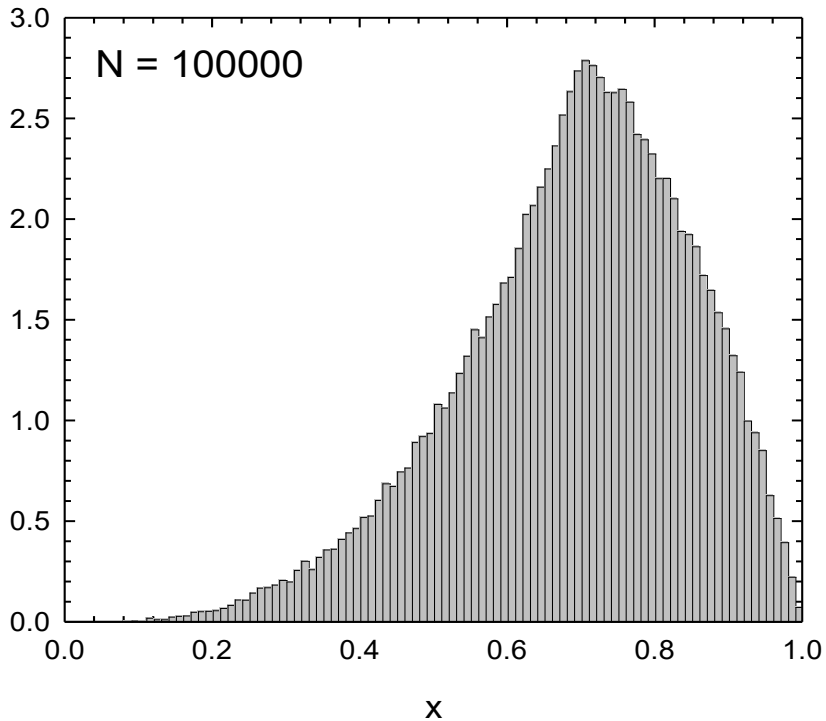
$$a = 2x$$

$$g(a) = \frac{1}{2} f\left(\frac{a}{2}\right)$$

$$g(a) = \begin{cases} a & 0 \le a \le 1 \\ 2 - a & 1 < a \le 2 \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

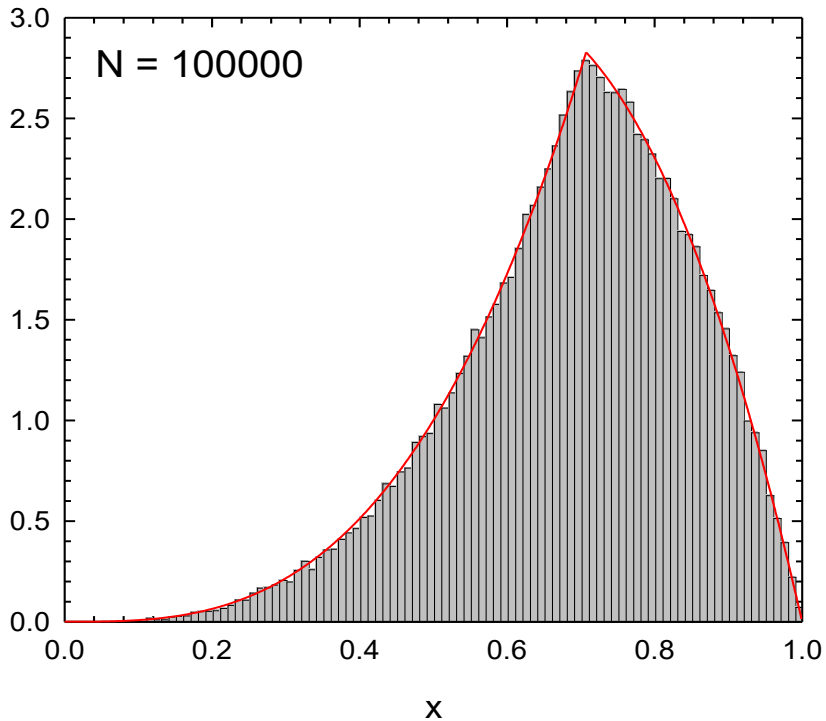
N = 100000
$\Delta = 0.01$



$$a = \sqrt{x}$$

Funkce náhodné proměnné

N = 100000
$\Delta = 0.01$



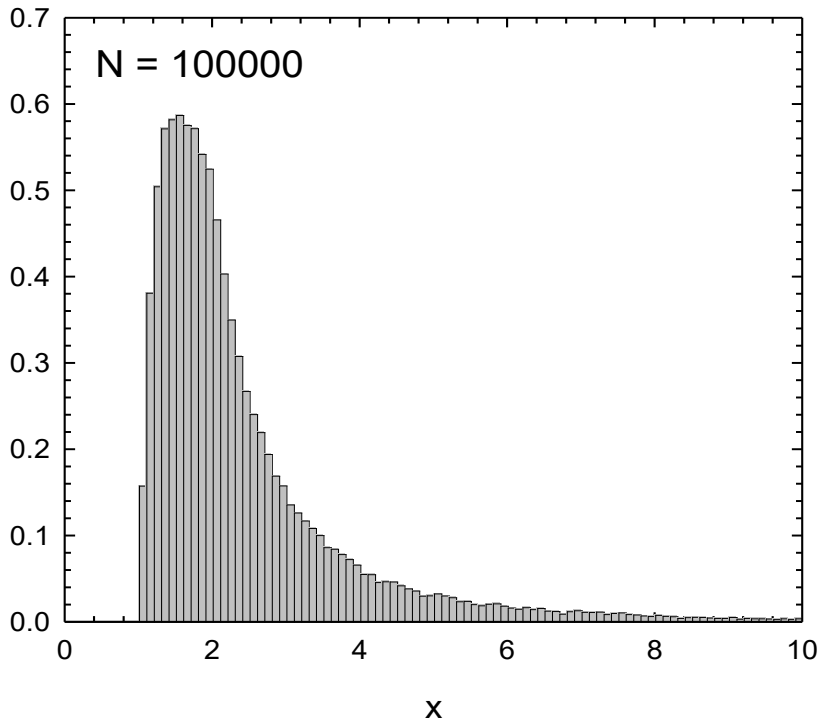
$$a = \sqrt{x}$$

$$g(a) = 2f\left(a^2\right)|a|$$

$$g(a) = \begin{cases} 8a^3 & 0 \le a \le \dfrac{1}{\sqrt{2}} \\ 8a\left(1 - a^2\right) & \dfrac{1}{\sqrt{2}} < a \le 1 \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

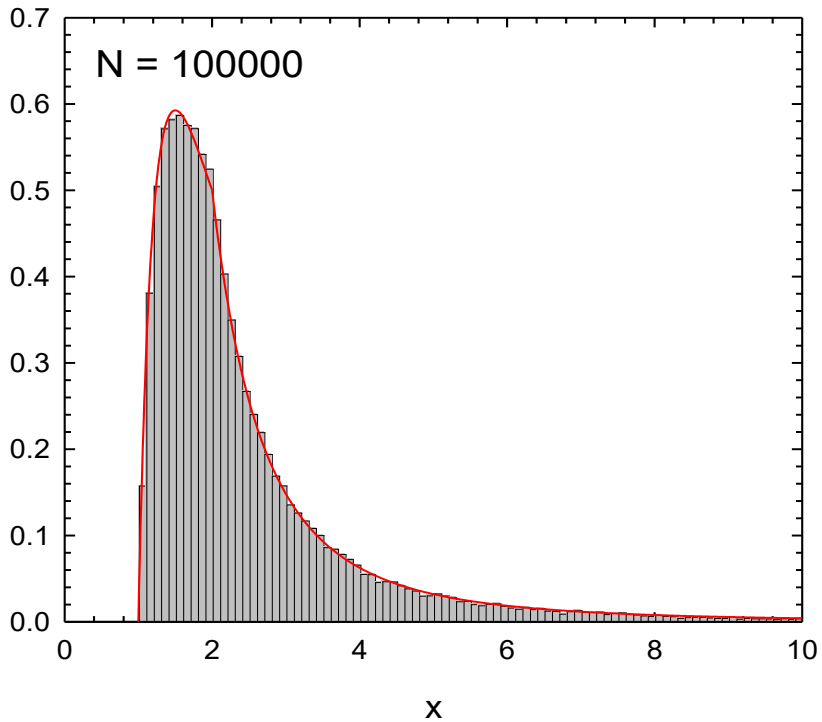
N = 100000
$\Delta = 0.10$



$$a = \frac{1}{x}$$

Funkce náhodné proměnné

N = 100000
$\Delta = 0.10$



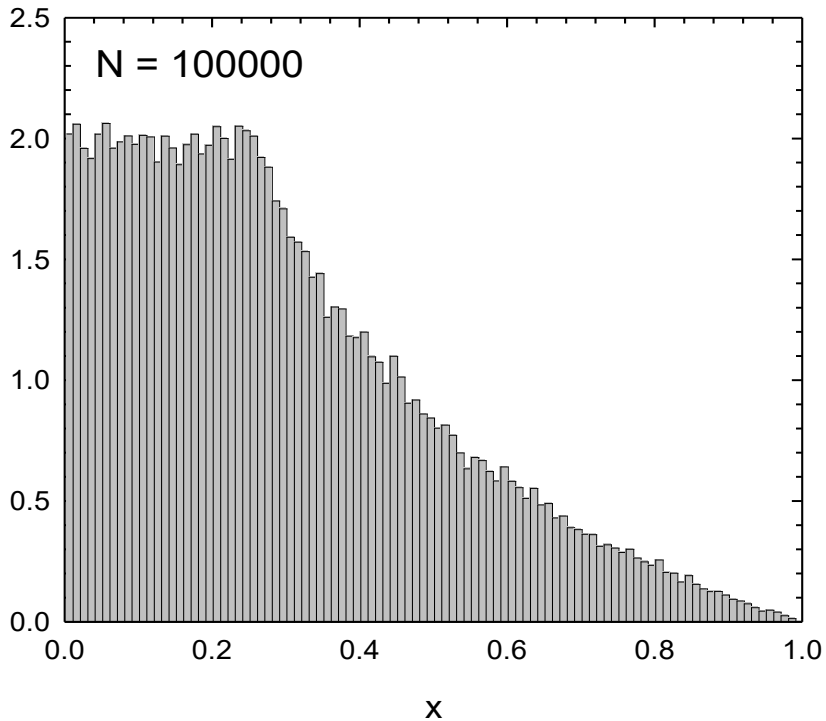
$$a = \frac{1}{x}$$

$$g(a) = f\left(\frac{1}{a}\right)\frac{1}{a^2}$$

$$g(a) = \begin{cases} \dfrac{4}{a^2}\left(1 - \dfrac{1}{a}\right) & 1 \le a \le 2 \\ \dfrac{4}{a^3} & 2 < a < \infty \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

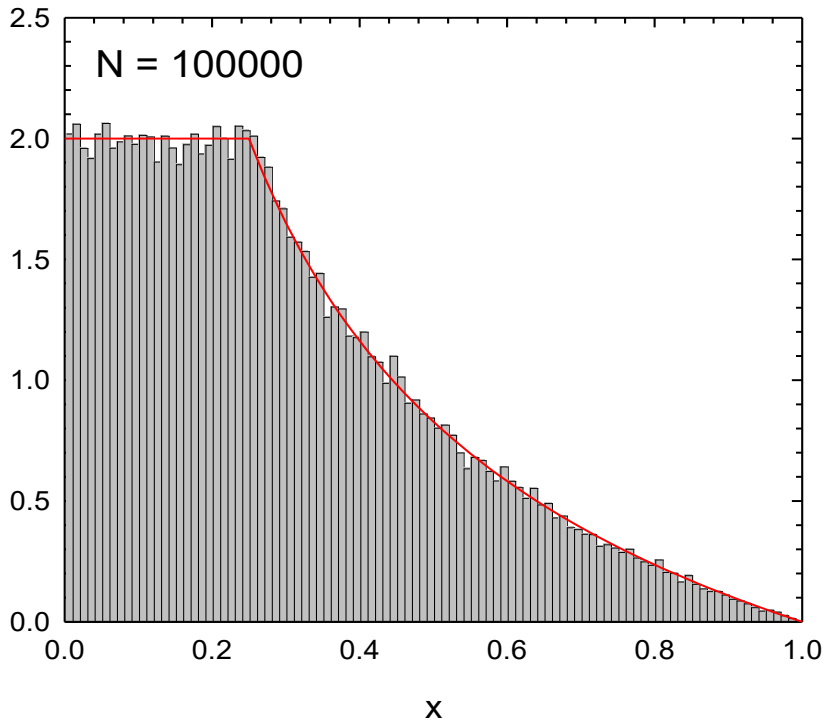
N = 100000
$\Delta = 0.01$



$$a = x^2$$

Funkce náhodné proměnné

N = 100000
Δ = 0.01



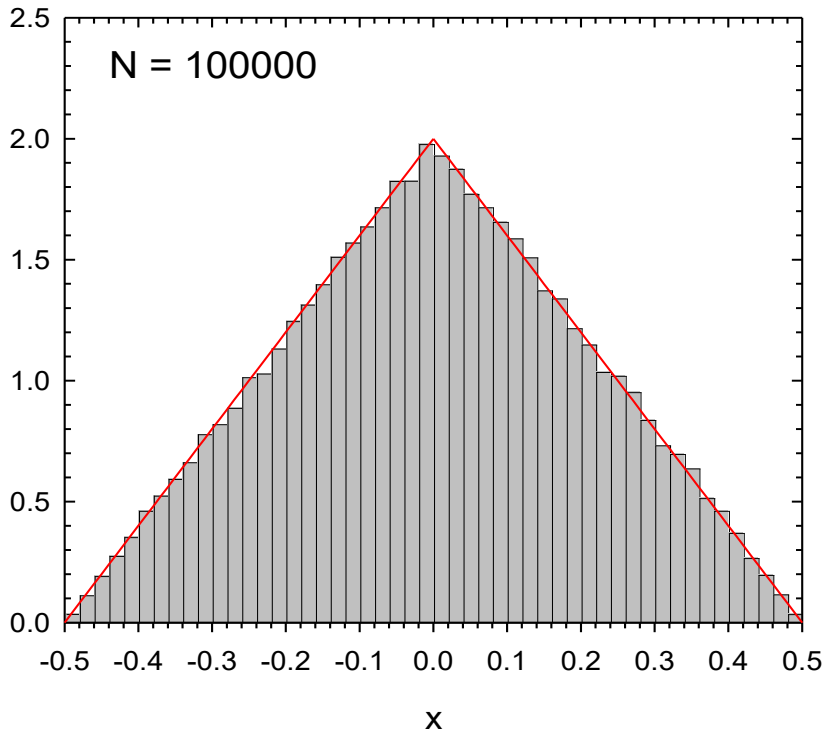
$$a = x^2$$

$$g(a) = \frac{f\left(\sqrt{a}\right) + f\left(-\sqrt{a}\right)}{2\sqrt{a}}$$

$$g(a) = \begin{cases} 2 & 0 \le a \le \frac{1}{4} \\ 2\left(\frac{1}{\sqrt{a}} - 1\right) & \frac{1}{4} < a \le 1 \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

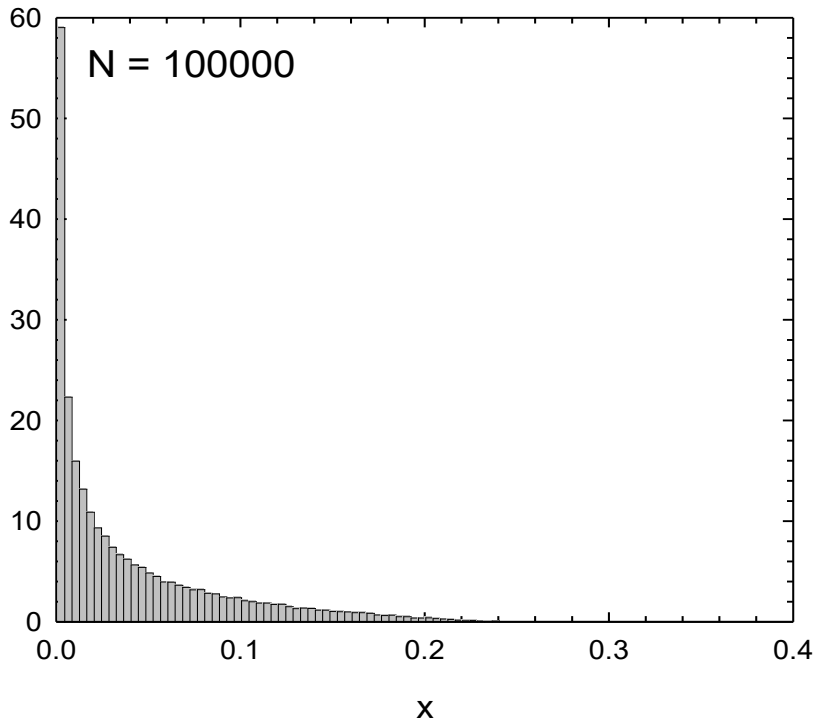
N = 100000
$\Delta = 0.01$



$$f(x) = \begin{cases} 4x + 2 & -\frac{1}{2} \le x \le 0 \\ -4x + 2 & 0 < x \le \frac{1}{2} \\ 0 & \text{jinak} \end{cases}$$

Funkce náhodné proměnné

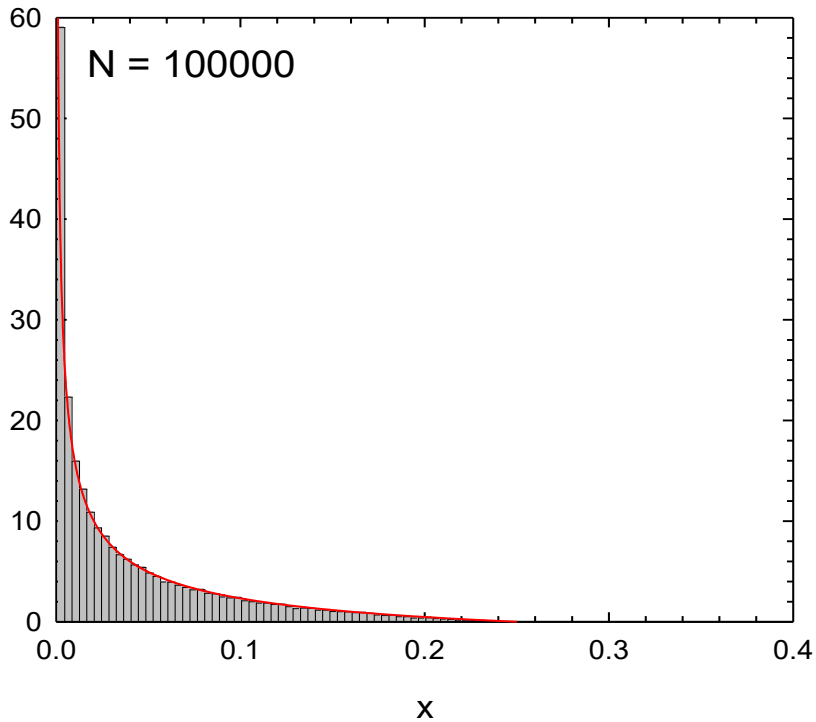
N = 100000
$\Delta = 0.004$



$$a = x^2$$

Funkce náhodné proměnné

N = 100000
$\Delta = 0.004$

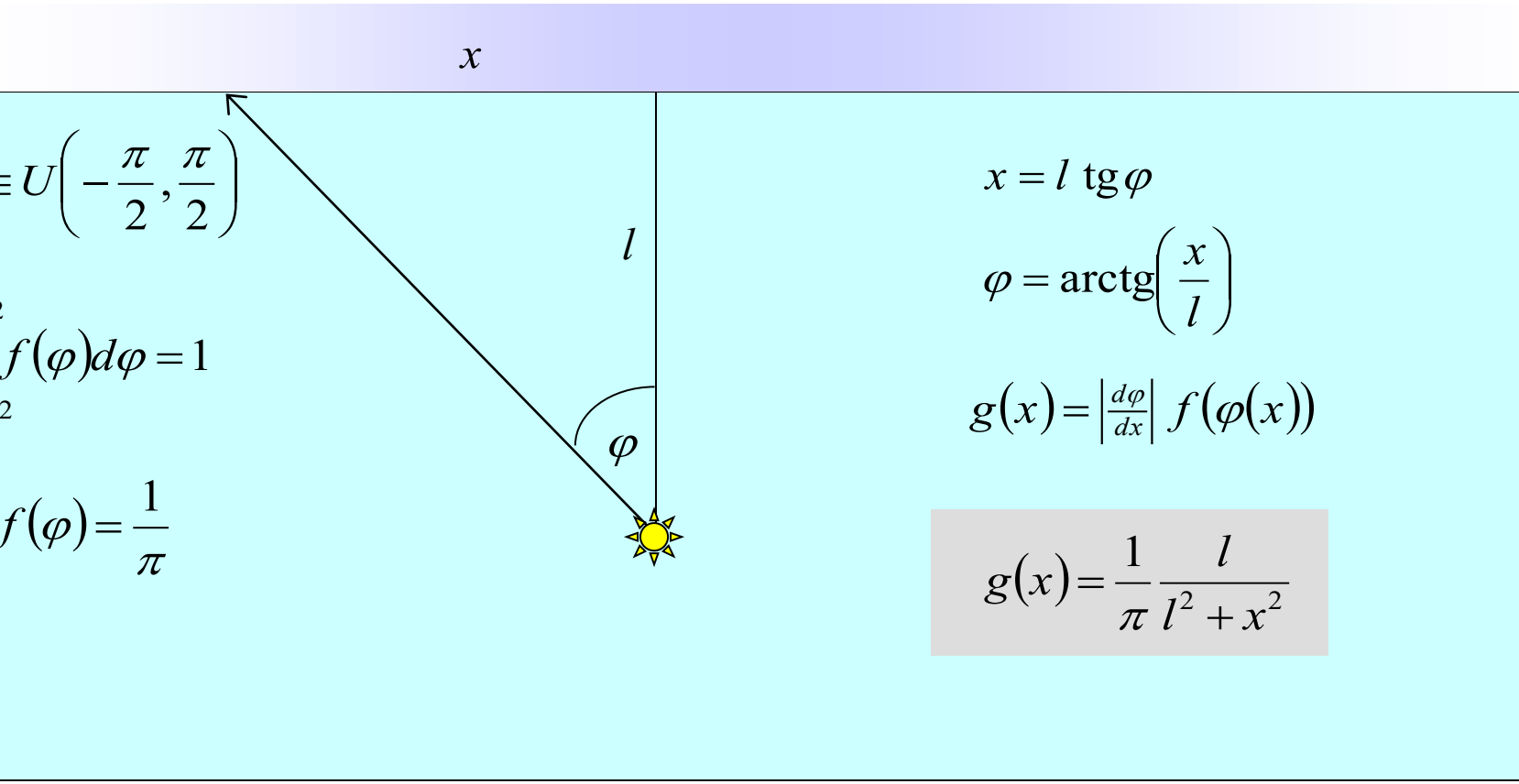


$$a = x^2$$

$$g(a) = \frac{f\left(\sqrt{a}\right) + f\left(-\sqrt{a}\right)}{2\sqrt{a}}$$

$$g(a) = \begin{cases} \dfrac{2}{\sqrt{a}} - 4 & 0 \le a \le \dfrac{1}{4} \\ 0 & \text{jinak} \end{cases}$$

Cauchyho rozdělení



$$\varphi \in U\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\int_{-\pi/2}^{\pi/2} f(\varphi)\, d\varphi = 1$$

$$f(\varphi) = \frac{1}{\pi}$$

$$x = l \operatorname{tg} \varphi$$

$$\varphi = \operatorname{arctg}\left(\frac{x}{l}\right)$$

$$g(x) = \left|\frac{d\varphi}{dx}\right| f(\varphi(x))$$

$$g(x) = \frac{1}{\pi} \frac{l}{l^2 + x^2}$$

Cauchyho rozdělení

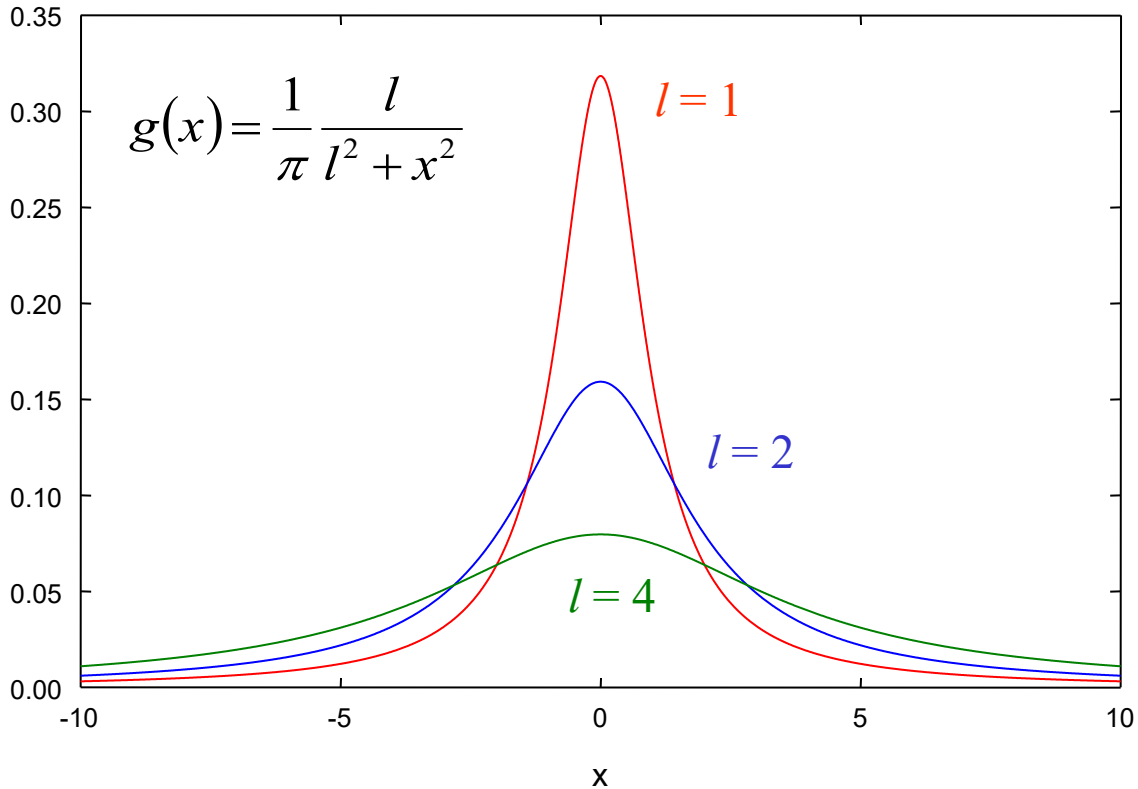
Cauchyho rozdělení

$$g(x) = \frac{1}{\pi\left(1+x^2\right)}$$

$$\gamma = 2l$$

Breit-Wignerovo rozdělení

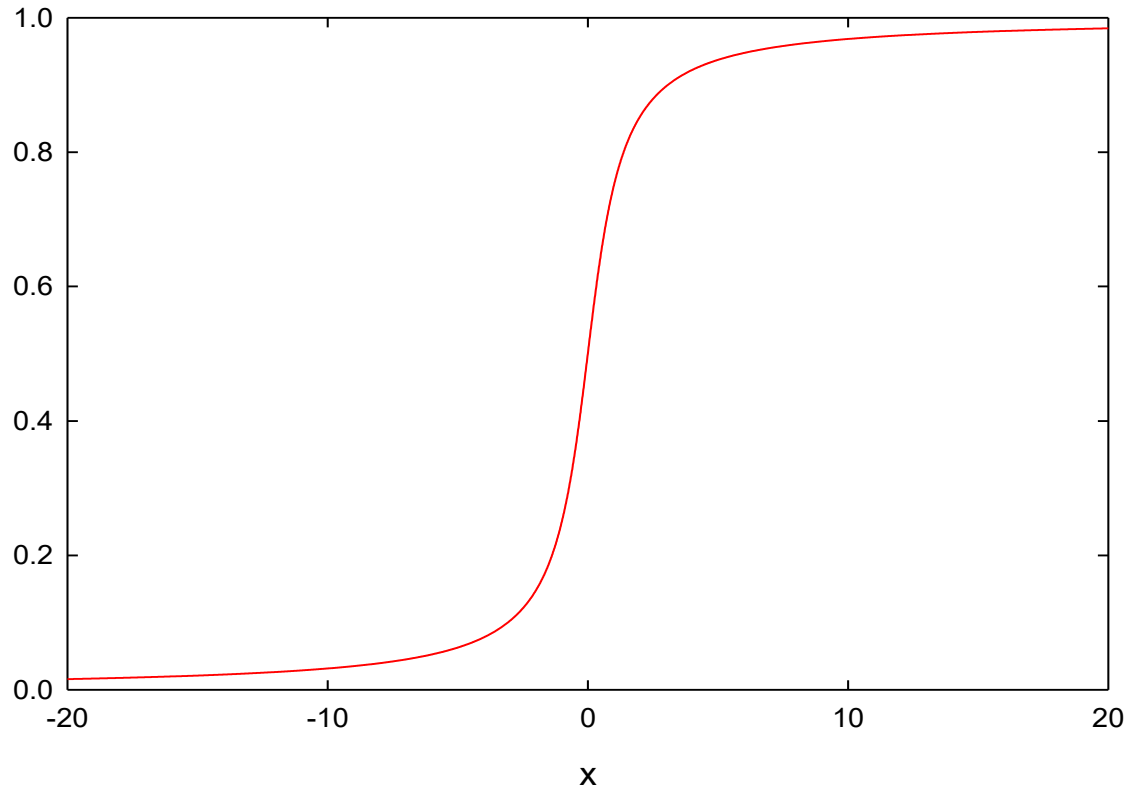
$$g(x) = \frac{1}{\pi}\frac{\gamma/2}{\gamma^2/4+\left(x-x_0\right)^2}$$



Cauchyho rozdělení

Distribuční funkce

$$F(x) = \frac{1}{\pi}\left(\operatorname{arctg} x + \frac{\pi}{2}\right)$$



Hustota pravděpodobnosti – případ dvou proměnných

marginální hustoty pravděpodobnosti

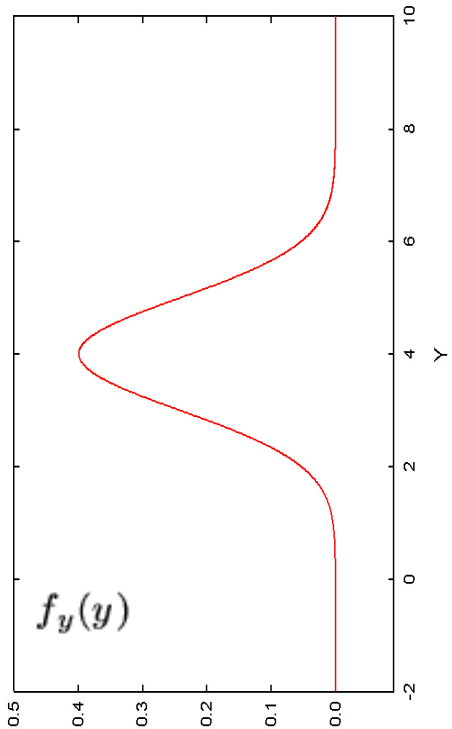
$$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

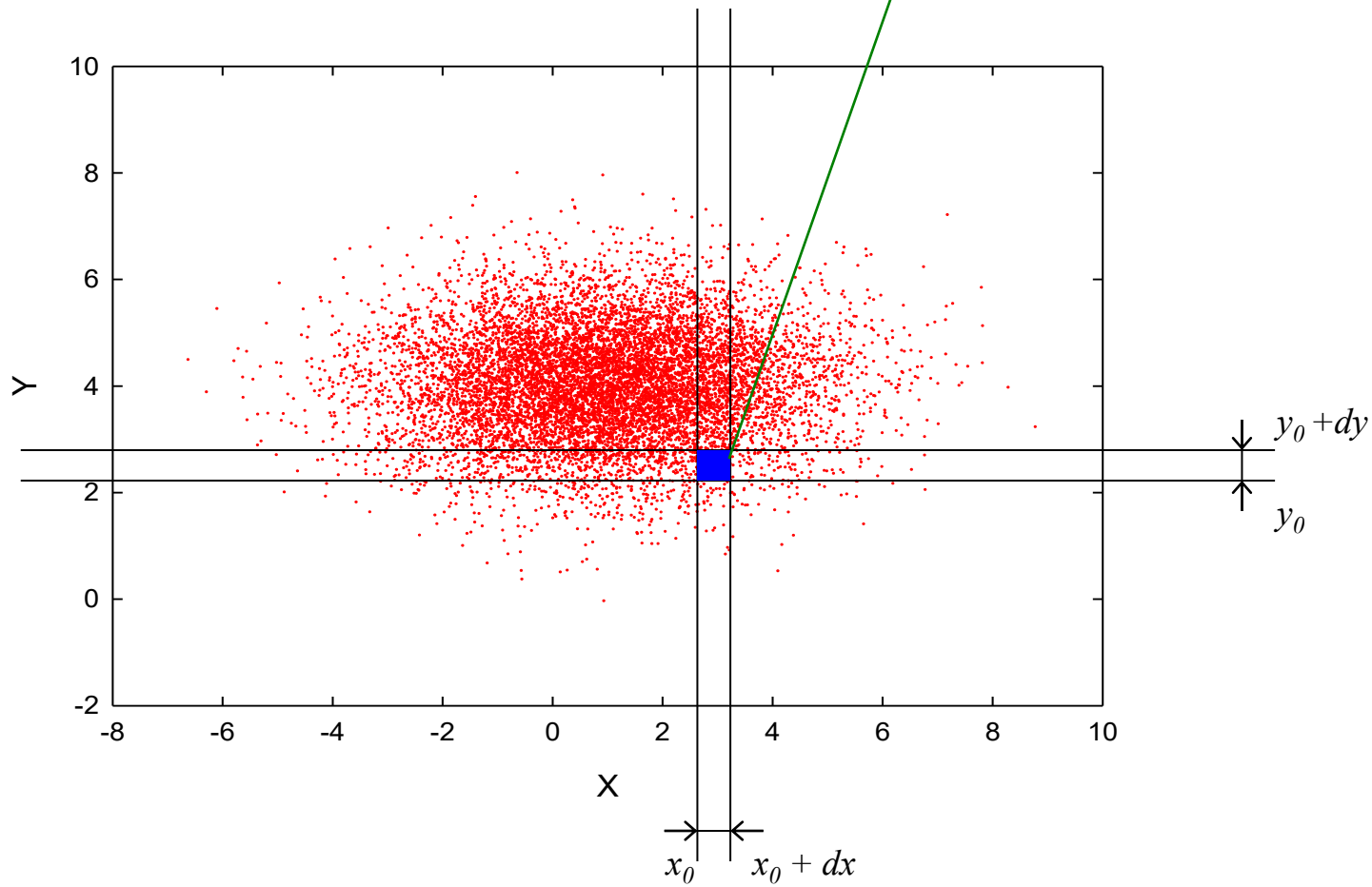
$$f_x(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$A: \quad x \in \langle x_0, x_0 + dx \rangle$$

$$B: \quad y \in \langle y_0, y_0 + dy \rangle$$

$$P(A \cap B) = f(x_0, y_0)\,dx\,dy$$





Vlnová funkce atomu vodíku

základní stav:

$$\Psi_{100}(r,\vartheta,\varphi) = R_{10}(r)Y_{00}(\vartheta,\varphi)$$

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}}\exp\left(-\frac{r}{a_0}\right)$$

$$Y_{00}(\vartheta,\varphi) = \frac{1}{\sqrt{4\pi}}$$

Bohrův poloměr

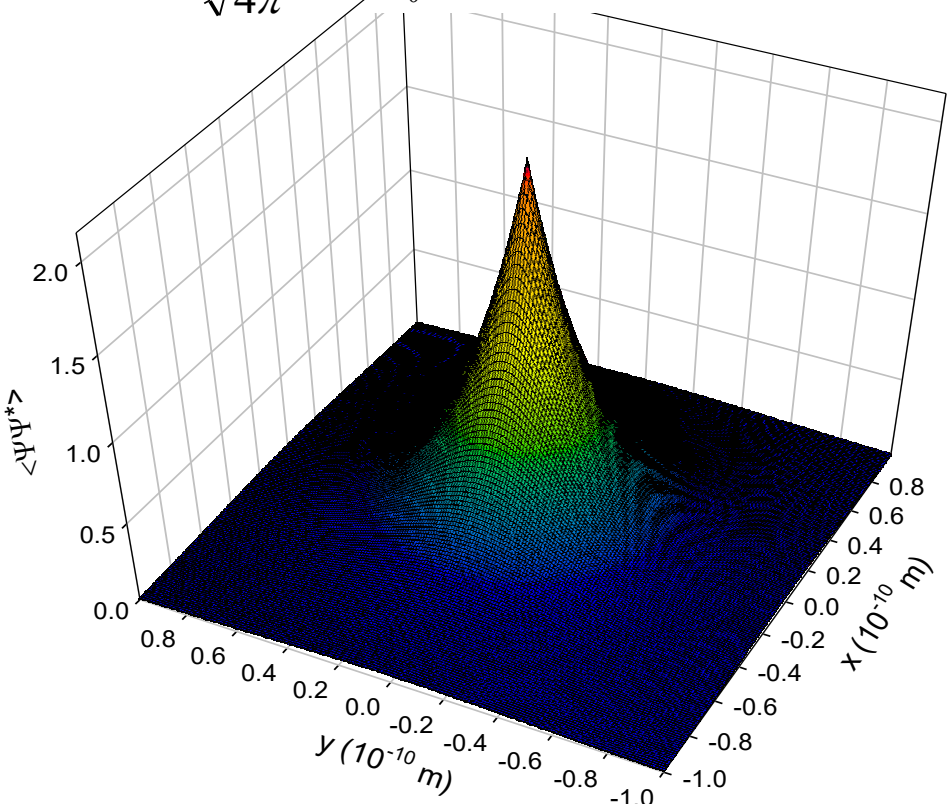
$a_0 = 0.529177$ Å

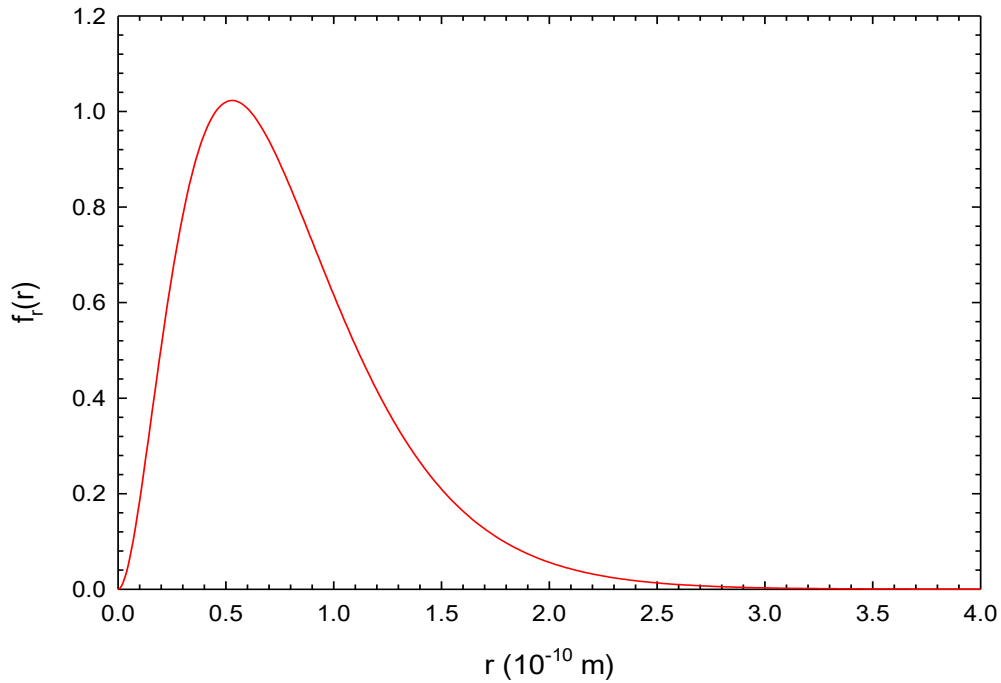
celková hustota pravděpodobnosti:

$$f(r,\vartheta,\varphi) = \Psi_{100}\Psi_{100}^* = \frac{1}{\pi a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

marginální hustota pravděpodobnosti:

$$f_r(r) = \int_0^{\pi}\int_0^{2\pi} f(r,\vartheta,\varphi)r^2\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi = \frac{4}{a_0^3}r^2\exp\left(-\frac{2r}{a_0}\right)$$

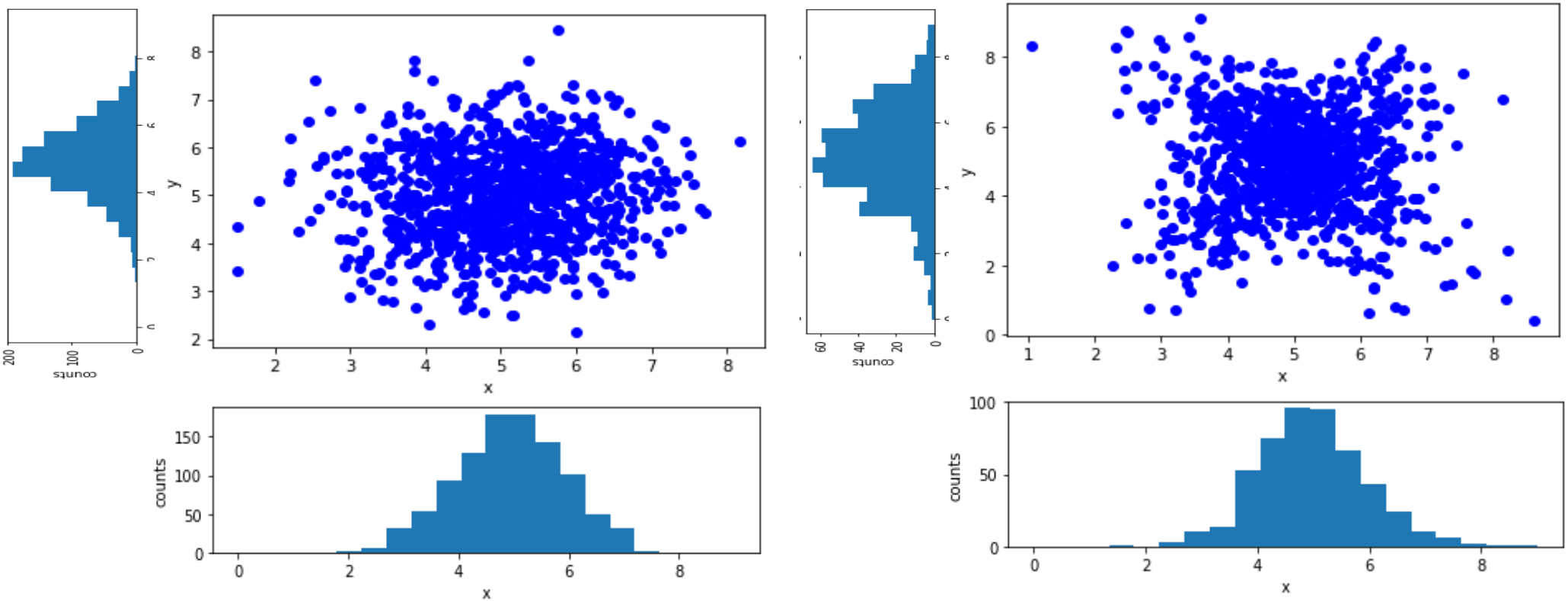




Hustota pravděpodobnosti – případ dvou proměnných

nezávislé náhodné proměnné

x, y jsou nezávislé $\Leftrightarrow$ $f(x,y) = f_x(x) f_y(y)$

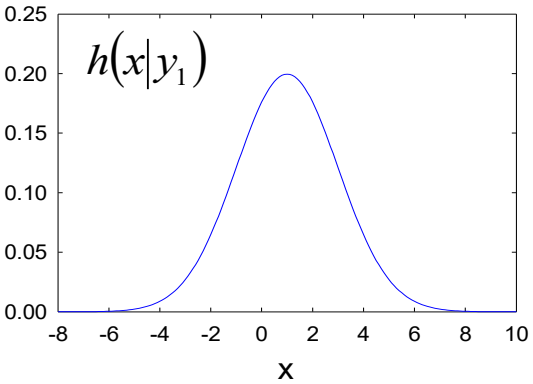


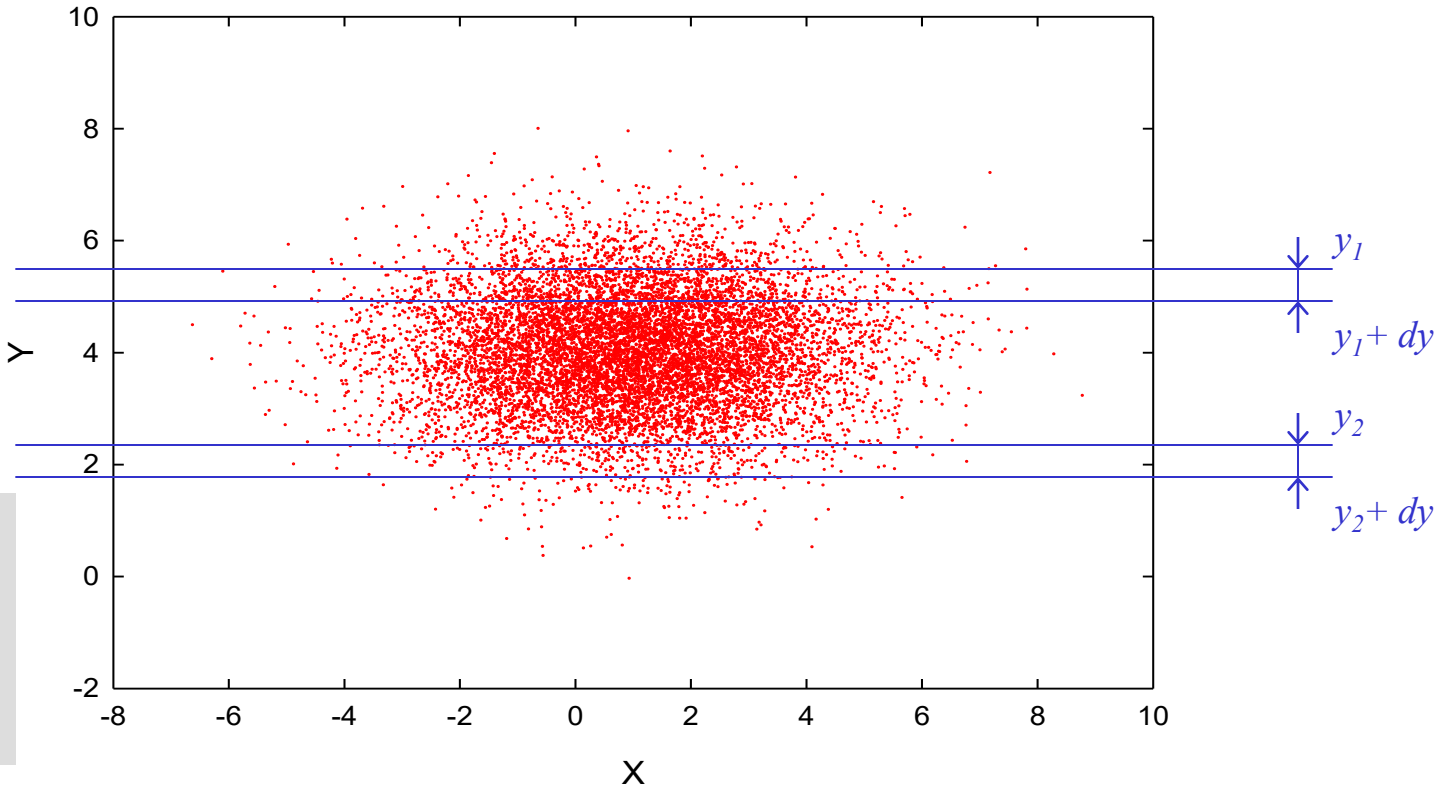
Podmíněná hustota pravděpodobnosti

$y \in \langle y_1, y_1 + dy \rangle$

podmíněná hustota pravděpodobnosti

$$h(x|y_1) = \frac{f(x, y_1)}{\int_{-\infty}^{\infty} f(x, y_1)\,dx} = \frac{f(x, y_1)}{f_y(y_1)}$$





Bayesův teorém:

$$h(x|y) = \frac{g(y|x) f_x(x)}{f_y(y)}$$