Laserový rezonátor - Gaussovský svazek

Gaussovský svazek

$$A \sim \frac{\exp(ikR)}{R}$$

$$A \sim \frac{\exp(ikq)}{q}$$

$$\tilde{A}(r,z) \sim \frac{1}{q}\exp\left(i\frac{kr^2}{2q}\right)\exp(ikz) \tag{1}$$

$$q = z - ib,\ b - Re,\ b > 0, \tag{2}$$

$$\widetilde{E}(r,z,t) = -iE_0 b\frac{1}{q}\exp\left(i\frac{kr^2}{2q}\right)\exp\left[-i(\omega t - kz)\right] \tag{3}$$

$$\frac{1}{q} = \frac{z}{z^2+b^2} + \frac{ib}{z^2+b^2} = \frac{1}{R(z)} + i\frac{2}{k}\frac{1}{w(z)^2} \tag{4}$$

$$R(z) = \frac{z^2+b^2}{z} \tag{5}$$

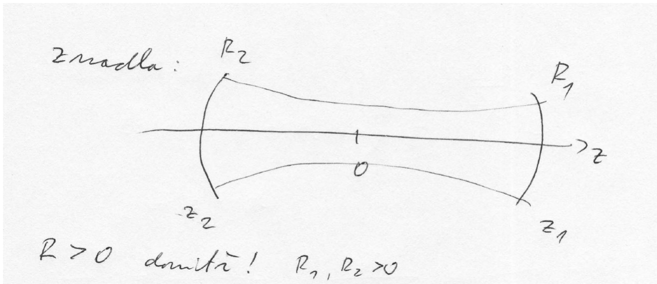
$$w(z)^2 = w_0^2\left(1+\frac{z^2}{b^2}\right) \tag{6}$$

$$w_0^2 = \frac{2}{k}b \tag{7}$$

$$-ib\frac{1}{q} = \frac{-ib}{z-ib} = \frac{1}{\left(1+\frac{z^2}{b^2}\right)^{\frac{1}{2}}}\exp\left[-i\Phi(z)\right] \tag{8}$$

$$\Phi(z) = \arctan\left(\frac{z}{b}\right) \tag{9}$$

$$\widetilde{E}(r,z,t) = E_0\frac{w_0}{w(z)}\exp\left(-\frac{r^2}{w(z)^2}\right)\exp\left(i\frac{kr^2}{2R(z)}\right)\exp\left[-i(\omega t - kz)\right]\exp\left[-i\Phi(z)\right]$$



y_{1}^{2}

$$
R_1 = \frac{b^2 + z_1^2}{z_1}
$$

$$
-R_2 = \frac{b^2 + z_2^2}{z_2} \tag{10}
$$

$$
z_1 - z_2 = L
$$

přímý výpočet

$$
z_1 = \frac{L\,(R_2 - L)}{R_1 + R_2 - 2L},\; z_2 = \frac{L\,(L - R_1)}{R_1 + R_2 - 2L} \tag{11}
$$

$$
b^2 = \frac{L\,(R_1 - L)\,(R_2 - L)\,(R_1 + R_2 - L)}{(R_1 + R_2 - 2L)^2} \tag{12}
$$

Podmínka gaussovského svazku: $b - \textit{reálné}$,.

$$
L\,(R_1 - L)\,(R_2 - L)\,(R_1 + R_2 - L) \geqq 0 \tag{13}
$$

$$
R_1 \left(1 - \frac{L}{R_1}\right) R_2 \left(1 - \frac{L}{R_2}\right) \frac{R_1 R_2}{L} \left(\frac{R_1 L}{R_1 R_2} + \frac{R_2 L}{R_1 R_2} - \frac{L^2}{R_1 R_2}\right) \geqq 0
$$

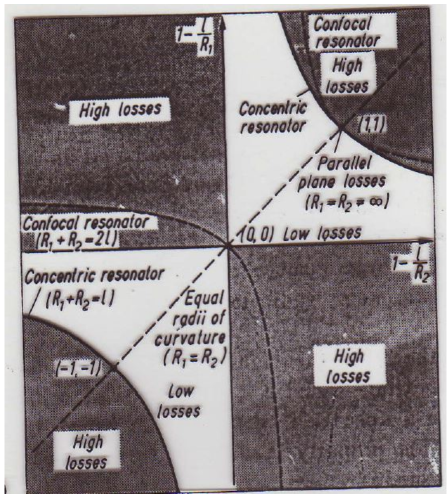
Zavedeme

$$
g_j = 1 - \frac{L}{R_j},\; j = 1, 2
$$

$$
g_1 g_2 \,(1 - g_1 g_2) \geqq 0
$$

$$
g_1 g_2 \geqq 0,\; g_1 g_2 \leqq 1 \tag{14}
$$

Boyd-Kogelnik



Vztah pro w

$$w(z)^2 = w_0^2\left(1+\frac{z^2}{b^2}\right)$$

$$w_0^2 = \frac{2}{k}b$$

$$1+\frac{z_1^2}{b^2} = \frac{R_1\left(R_1+R_2-2L\right)}{\left(R_1-L\right)\left(R_1+R_2-L\right)}$$

$$w(z_1)^2 = \frac{2R_1}{k}\left[\frac{L\left(R_2-L\right)}{\left(R_1-L\right)\left(R_1+R_2-L\right)}\right]^{\frac{1}{2}},\ w(z_2)^2 = \frac{2R_2}{k}\left[\frac{L\left(R_1-L\right)}{\left(R_2-L\right)\left(R_1+R_2-L\right)}\right]^{\frac{1}{2}} \quad (15)$$

Frekvenční podmínka:

$$kz_1 - \arctan\left(\frac{z_1}{b}\right) - \left[kz_2 - \arctan\left(\frac{z_2}{b}\right)\right] = n\pi,\ n - cel\acute{e} \quad (16)$$

$$\arctan\alpha - \arctan\beta = \arctan\frac{\alpha-\beta}{1+\alpha\beta}$$

$$kL - \arctan\left(\frac{Lb}{b^2+z_1z_2}\right) = n\pi$$

$$kL - \arctan\left(\frac{L\left(R_1+R_2-L\right)}{\left(R_1-L\right)\left(R_2-L\right)}\right)^{\frac{1}{2}} = n\pi$$

$$kL - \arctan\left(\frac{1-g_1g_2}{g_1g_2}\right)^{\frac{1}{2}} = n\pi$$

Pozn.

$$g_1g_2 = y$$

$$\frac{1-y}{y} = \frac{1}{x^2}$$

$$\arctan\frac{1}{x} = \arccos\frac{x}{\sqrt{1+x^2}}$$

$$\arctan\left[\frac{1-y}{y}\right]^{\frac{1}{2}} = \arccos\left[\frac{x^2}{1+x^2}\right]^{\frac{1}{2}} = \arccos\sqrt{y}$$

$$kL - \arccos\sqrt{g_1 g_2} = n\pi \tag{17}$$

Rezonátor konfokální $R_1 = R_2 = L$. $b = \frac{L}{2}$. $w_0^2 = \frac{2}{k}\frac{L}{2}$. $w(z_1, z_2)^2 = 2w_0^2 . g_1 = g_2 = 0$. $\arccos(g_1 g_2) = (2p+1)\frac{\pi}{2} . \nu = n\frac{c}{2L} + (2p+1)\frac{c}{4L}$.

Rezonátor rovinný $g_1 = g_2 = 1$

Hemisférický $R_2 = \infty R_1 = L + \Delta L$,$z_1 = L$, $R_1 = \frac{b^2+z_1^2}{z_1^2}$,$b^2 = L\triangle L . w_0^2 = \frac{\lambda L}{\pi}\sqrt{\frac{\triangle L}{L}}$.

Gaussovský svazek- matice

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$$q' = \frac{Aq+B}{Cq+D}$$

Stabilní rezonátor:

$$q' = q$$

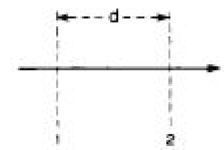
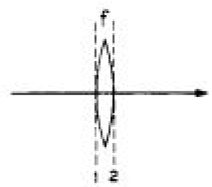
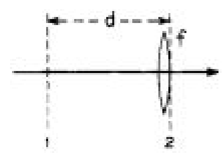
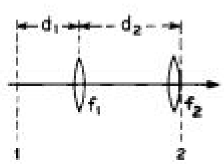
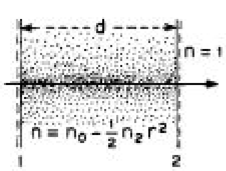
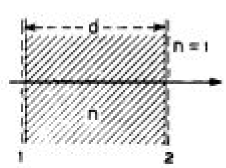
$$q = \frac{A - D \pm \sqrt{(A-D)^2 + 4BC}}{2C}$$

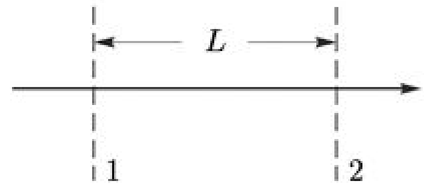
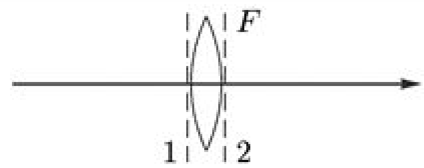
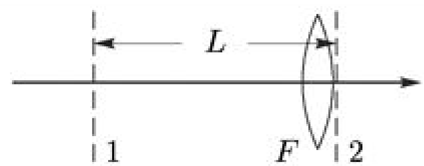
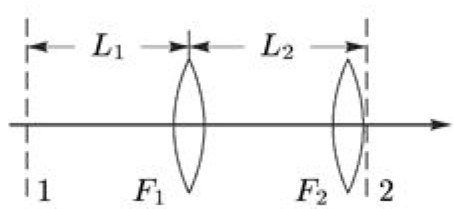
chceme $q = z - ib$, $b > 0$, $\sqrt{\ } = i\sqrt{4 - (A+D)^2}$. Podmínka

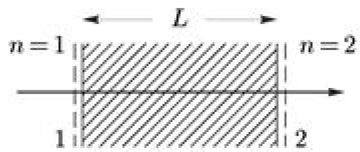
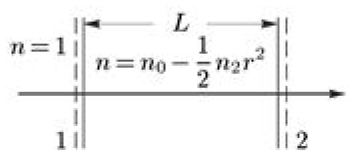
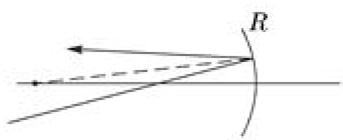
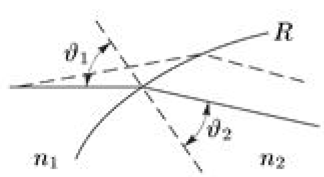
$$(A+D)^2 \leqq 4$$

$$\frac{|A+D|}{2} \leqq 1$$

stejně jako geometrická optika ...

NO	OPTICAL SYSTEM	RAY TRANSFER MATRIX
1		$\begin{vmatrix} 1 & d \\ 0 & 1 \end{vmatrix}$
2		$\begin{vmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{vmatrix}$
3		$\begin{vmatrix} 1 & d \\ -\frac{1}{f} & 1-\frac{d}{f} \end{vmatrix}$
4		$\begin{vmatrix} 1-\frac{d_2}{f_1} & d_1+d_2-\frac{d_1 d_2}{f_1} \\ -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d_2}{f_1 f_2} & 1-\frac{d_1}{f_1}-\frac{d_2}{f_2}-\frac{d_1}{f_2}+\frac{d_1 d_2}{f_1 f_2} \end{vmatrix}$
5		$\begin{vmatrix} \cos d\sqrt{\frac{n_2}{n_0}} & \frac{1}{\sqrt{n_0 n_2}}\sin d\sqrt{\frac{n_2}{n_0}} \\ -\sqrt{n_0 n_2}\,\sin d\sqrt{\frac{n_2}{n_0}} & \cos d\sqrt{\frac{n_2}{n_0}} \end{vmatrix}$
6		$\begin{vmatrix} 1 & d/n \\ 0 & 1 \end{vmatrix}$

№	Оптическая система	Лучевая матрица
1		$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
2		$\begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix}$
3		$\begin{pmatrix} 1 & L \\ -1/F & 1 - L/F \end{pmatrix}$
4		$\begin{pmatrix} 1 - \frac{L_2}{F_1} & L_1 + L_2 - \frac{L_1 L_2}{F_2} \\ -\frac{1}{F_1} - \frac{1}{F_2} + \frac{L_2}{F_1 F_2} & 1 - \frac{L_1}{F_1} - \frac{L_2}{F_2} + \frac{L_1 L_2}{F_1 F_2} \end{pmatrix}$

№	Оптическая система	Лучевая матрица
5		$\begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}$
6		$\begin{pmatrix} \cos L\sqrt{\frac{n_2}{n_0}} & \frac{\sin L\sqrt{\frac{n_2}{n_0}}}{\sqrt{n_0 n_2}} \\ -\sqrt{n_0 n_2}\,\sin L\sqrt{\frac{n_2}{n_0}} & \cos L\sqrt{\frac{n_2}{n_0}} \end{pmatrix}$
7		$\begin{pmatrix} -1 & 0 \\ 2/R & -1 \end{pmatrix}$
8		в плоскости падения $\begin{pmatrix} 1 & 0 \\ 2/(R\cos\gamma) & 1 \end{pmatrix}$
9		в плоскости, перпендикулярной к плоскости падения $\begin{pmatrix} 1 & 0 \\ 2\cos\gamma/R & 1 \end{pmatrix}$
10		в плоскости падения $\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{n_2}{n_1}$ $\begin{pmatrix} n_2\,\frac{\cos\vartheta_2}{\cos\vartheta_1} & 0 \\ \frac{n_1\cos\vartheta_1 - n_2\cos\vartheta_2}{R\cos\vartheta_1\cos\vartheta_2} & n_1\,\frac{\cos\vartheta_1}{\cos\vartheta_2} \end{pmatrix}$
11		в плоскости, перпендикулярной к плоскости падения $\begin{pmatrix} n_2 & 0 \\ \frac{n_1\cos\vartheta_1 - n_2\cos\vartheta_2}{R} & n_1 \end{pmatrix}$, $\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{n_2}{n_1}$