Fourierova optika

Fourierova transformace

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} g(\omega)\exp(-i\,\omega t)\,d\omega\,,$$

$$g(\omega)=\int_{-\infty}^{\infty} f(t)\exp(+i\,\omega t)dt$$

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} g(k)\exp(ik\,x)dk \qquad g(k)=\int_{-\infty}^{\infty} f(x)\exp(-i\,kx)dx$$

Rozvoj do rovinných vln

$$f(x,y)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} g(k_x,k_y)\exp[i(k_x x+k_y y)]\,dk_x dk_y$$

$$g(k_x,k_y)=\int_{-\infty}^{\infty} f(x,y)\exp[-i(k_x x+k_y y)]\,dx\,dy$$

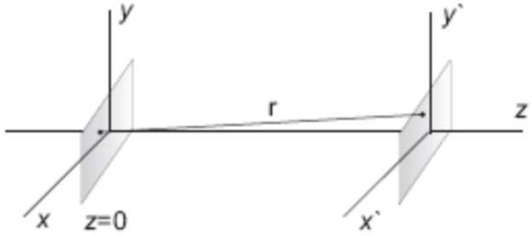
$$k_x=\frac{2\pi}{\lambda_x},\quad k_y=\frac{2\pi}{\lambda_y}$$

Prostorové frekvence

Prostorové periody

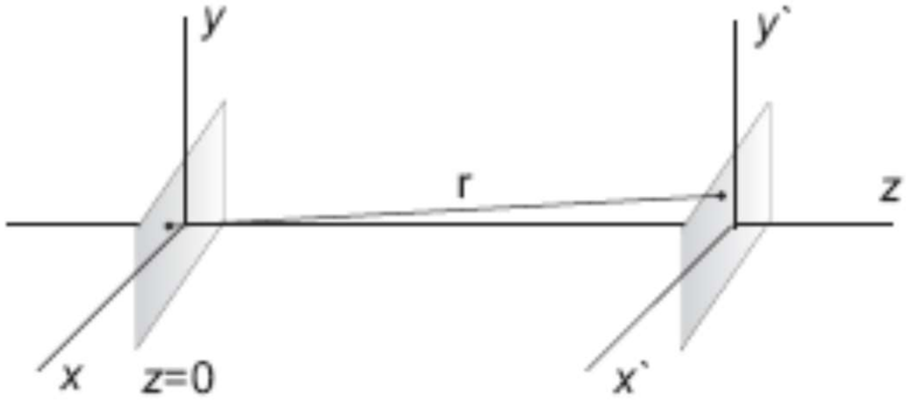
Fresnelova a Fraunhoferova aproximace

Známé pole E v z=0



$$\tilde{E}(x', y') = \frac{-i}{\lambda} A \int_{APERTURA} F(\theta) \frac{1}{r\, r_z} \exp\left[i k (r + r_z)\right] dS$$

$$\tilde{E}(x', y') = \frac{-i}{\lambda} \int_{APERTURA} \tilde{E}_S(x, y) \frac{1}{r} \exp(i k r)\, dx\, dy$$



$$r = \sqrt{z^2 + (x - x')^2 + (y - y')^2} \approx z + \frac{(x - x')^2 + (y - y')^2}{2z} \cdot$$

PARAXIÁLNÍ APROXIMACE:

$$F = 1, r \rightarrow z \ \ ve \ \ jmenovateli$$

$$\tilde{E}(x', y') =$$

$$= \frac{-i}{\lambda} \frac{1}{z} \exp(ikz) \int_{APERTURA} \tilde{E}_S(x, y) \exp\left\{\frac{ik}{2z}\left[(x - x')^2 + (y - y')^2\right]\right\} dx\, dy\,,$$

$$\tilde{E}(x', y') = \frac{-i}{\lambda} \frac{1}{z} \exp(ikz) \exp\left[\frac{ik}{2z}\left(x'^2 + y'^2\right)\right].$$

$$\int_{APERTURA} \tilde{E}_S(x, y) \exp\left[\frac{ik}{2z}\left(x^2 + y^2\right)\right] \exp\left[\frac{-ik}{z}(xx' + yy')\right] dx\, dy\,.$$

Fresnelova paraxiální aproximace

$$\tilde{E}(x',y') =$$

$$= \frac{-i}{\lambda}\frac{1}{z}\exp(ikz) \int_{APERTURA} \tilde{E}_S(x,y)\exp\left\{\frac{ik}{2z}\left[(x-x')^2+(y-y')^2\right]\right\} dx\,dy\,,$$

$$\tilde{E}(x',y') = \frac{-i}{\lambda}\frac{1}{z}\exp(ikz)\exp\left[\frac{ik}{2z}\left(x'^2+y'^2\right)\right].$$

$$\int_{APERTURA} \tilde{E}_S(x,y)\exp\left[\frac{ik}{2z}\left(x^2+y^2\right)\right]\exp\left[\frac{-ik}{z}\left(xx'+yy'\right)\right] dx\,dy\,.$$

= 0

Fresnelova paraxiální aproximace

Fraunhoferova aproximace

$$\tilde{E}(x', y') =$$

$$= \frac{-i}{\lambda}\frac{1}{z}\exp(ikz)\exp\left[\frac{ik}{2z}\left(x'^2 + y'^2\right)\right]\int_{APERTURA}\tilde{E}_S(x, y)\exp\left[\frac{-ik}{z}(xx' + yy')\right]dx\,dy$$

$$k_x = \frac{k\,x'}{z},\ k_y = \frac{k\,y'}{z}.$$

Zklenutí pole, pro body blízko osy možno brát konst.

$$\tilde{E}(x', y') = \tilde{E}(k_x, k_y) = \tilde{A}\int_{APERTURA}\tilde{E}_S(x, y)\exp\left[-i\left(k_x\,x + k_y\,y\right)\right]dx\,dy$$

Prostorové frekvence $k_x = \frac{k\,x'}{z},\ k_y = \frac{k\,y'}{z}.$

$$\tilde{E}(x',y')=$$

$$=\frac{-i}{\lambda}\frac{1}{z}\exp(ikz)\exp\left[\frac{ik}{2z}(x'^2+y'^2)\right]\int_{APERTURA}\tilde{E}_S(x,y)\exp\left[\frac{-ik}{z}(xx'+yy')\right]dx\,dy$$

$$k_x=\frac{kx'}{z},\quad k_y=\frac{ky'}{z}.$$

Zklenutí pole, pro body blízko osy možno brát konst.

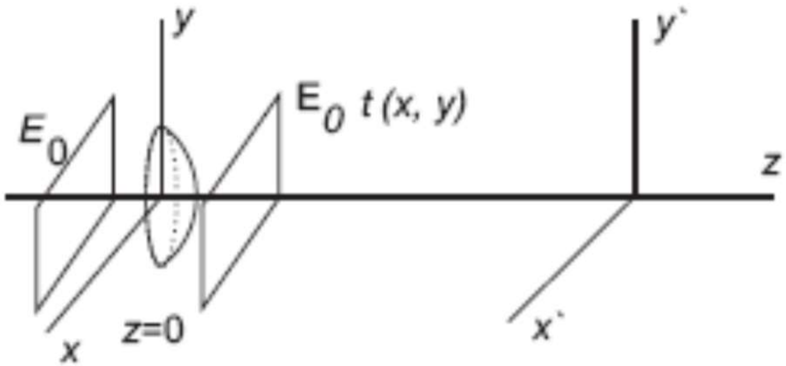
$$\tilde{E}(x',y')=\tilde{E}(k_x,k_y)=\tilde{A}\int_{APERTURA}\tilde{E}_S(x,y)\exp\left[-i(k_x x+k_y y)\right]dx\,dy$$

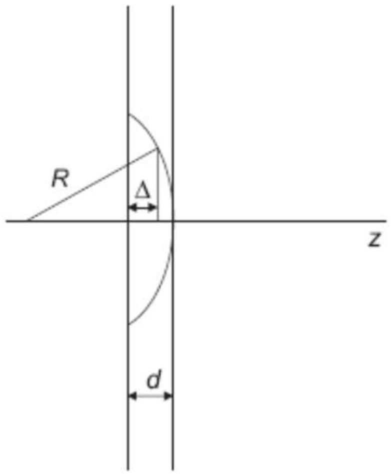
Fourierova transformace

Prostorové frekvence $k_x=\frac{kx'}{z},\quad k_y=\frac{ky'}{z}.$

Daleké pole (abychom mohli zanedbat ten člen s kvadráty x´, y´) nutné pro Fraunhoferovu aproximaci.

Co teď **čočka??**





$$\tilde{E}_S(x,y)=\tilde{E}_0\,\tilde{t}(x,y)$$

$$\tilde{t}(x,y)=\exp[i\,\varphi(x,y)]$$

$$\varphi(x,y)=k_0\,d+(n-1)k_0\,\Delta(x,y)$$

$$\Delta(x,y)=d-\left(R-\sqrt{R^2-x^2-y^2}\right).$$

$$\Delta(x,y)=d-R\left(1-\sqrt{1-\frac{x^2+y^2}{R^2}}\right)$$

$$x^2+y^2 \ll R^2$$

$$\sqrt{1-\frac{x^2+y^2}{R^2}}\approx 1-\frac{x^2+y^2}{2R^2}$$

$$\Delta(x,y)=d-\frac{x^2+y^2}{2R}$$

$$\tilde{t}(x, y) = \exp(i\varphi_0)\exp\left(-i k_0 \frac{x^2 + y^2}{2f'}\right)$$

kde

$$\frac{1}{f'} = \frac{n-1}{R}.$$

$$\tilde{E}(x', y') =$$

$$= \frac{-i}{\lambda z} \exp(i k z + i \varphi_0) \exp\left(ik \frac{x'^2 + y'^2}{2z} \right)$$

$$\iint_{APERTURA} \tilde{E}_0 \exp\left[i k \frac{x^2 + y^2}{2} \left(\frac{1}{z} - \frac{1}{f'} \right) \right] \exp\left[-\frac{i k}{z} (x x' + y y') \right] dx\, dy$$

Osvětlení čočky může být modulované $\tilde{E}_0 = \tilde{E}_0 (x, y)$

V ohniskové rovině čočky $z = f'$

$$\tilde{E}(x', y') = \tilde{E}(k_x, k_y) = \tilde{B} \int_{APERTURA} \tilde{E}_S (x, y) \exp\left[-i (k_x x + k_y y) \right] dx\, dy$$

$$\tilde{E}(x', y') =$$

$$= \frac{-i}{\lambda z} \exp(i k z + i \varphi_0) \exp\left(ik \frac{x'^2 + y'^2}{2 z} \right)$$

$$\iint\limits_{APERTURA} \tilde{E}_0 \exp\left[i k \frac{x^2 + y^2}{2} \left(\frac{1}{z} - \frac{1}{f'} \right) \right] \exp\left[-\frac{i k}{z} (x x' + y y') \right] dx\, dy$$

vynuluje se

Osvětlení čočky může být modulované $\tilde{E}_0 = \tilde{E}_0(x, y)$

V ohniskové rovině čočky $z = f'$

$$\tilde{E}(x', y') = \tilde{E}(k_x, k_y) = \tilde{B} \int\limits_{APERTURA} \tilde{E}_S(x, y) \exp\left[-i \left(k_x x + k_y y \right) \right] dx\, dy$$

$$\tilde{E}(x', y') = \tilde{E}(k_x, k_y) = \tilde{B} \int_{APERTURA} \tilde{E}_S(x, y) \exp\left[-i\left(k_x x + k_y y\right)\right] dx\, dy$$

Opět Fourierova transformace !

Teď bez ohledu na daleké pole !!!

Čili pro ohniskové roviny čoček možno používat Fraunhoferovu difrakci !!!!!!

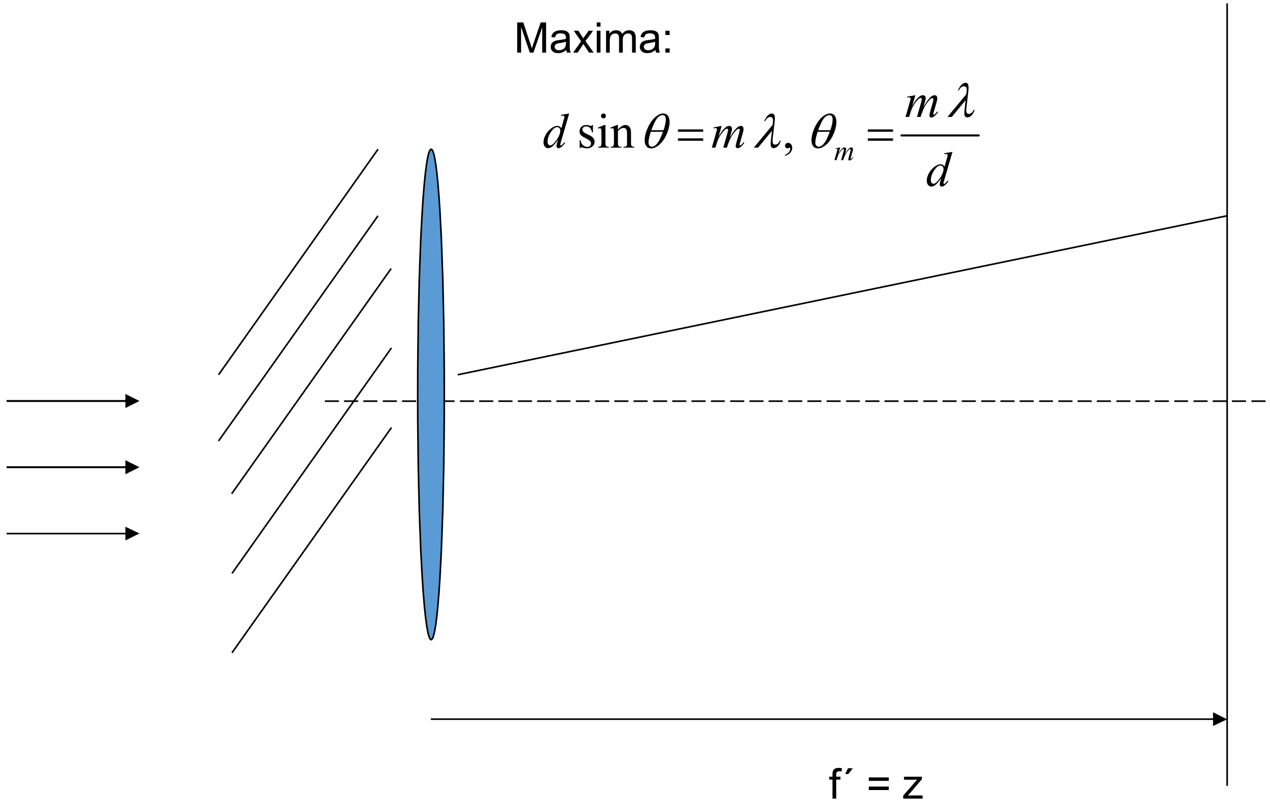
Například: teleskop, mikroskop, oko, apod.

Prostorová filtrace

Příklad optického zpracování obrazu

Maxima:

$$d \sin\theta = m\lambda,\ \theta_m = \frac{m\lambda}{d}$$



$$\theta_m = {y'_m}/{f'}$$

$$y'_m = m\frac{\lambda f'}{d}$$

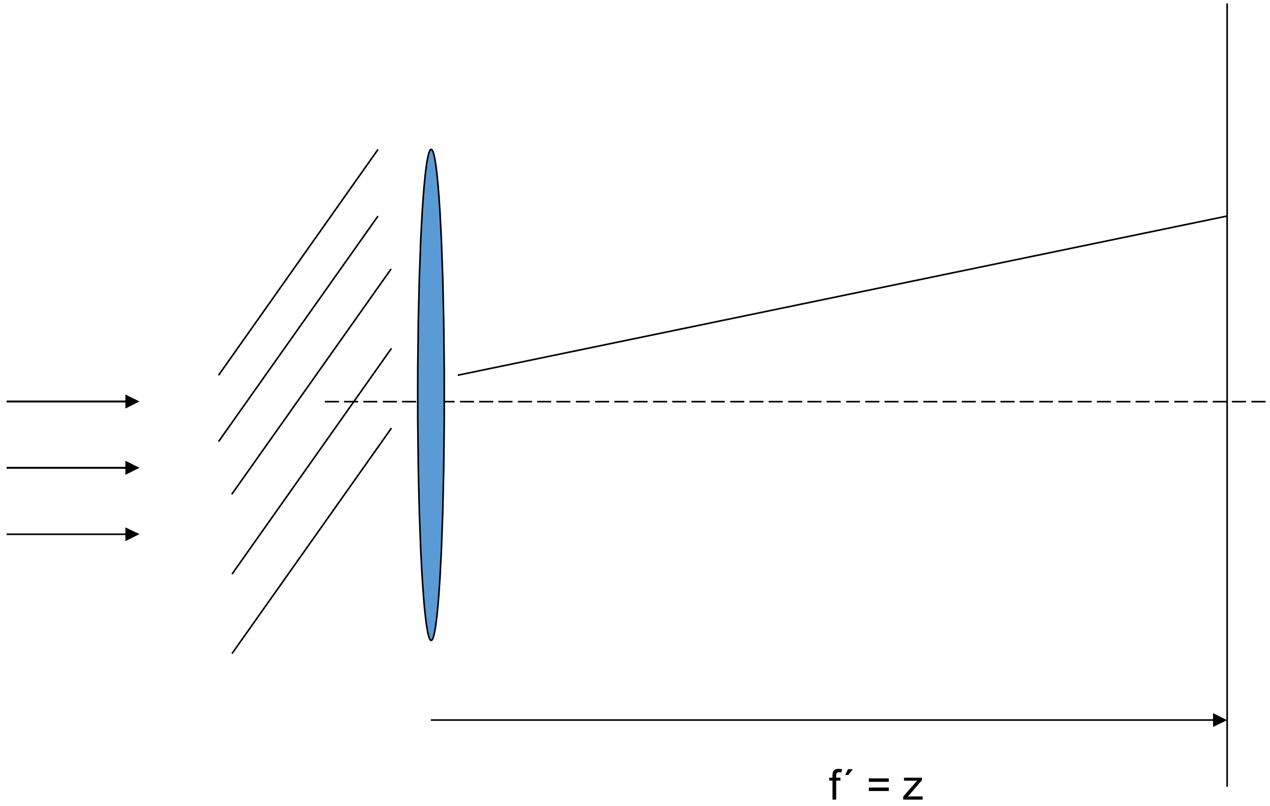
Prostorová frekvence

$$k_y = \frac{k\,y'}{z}$$

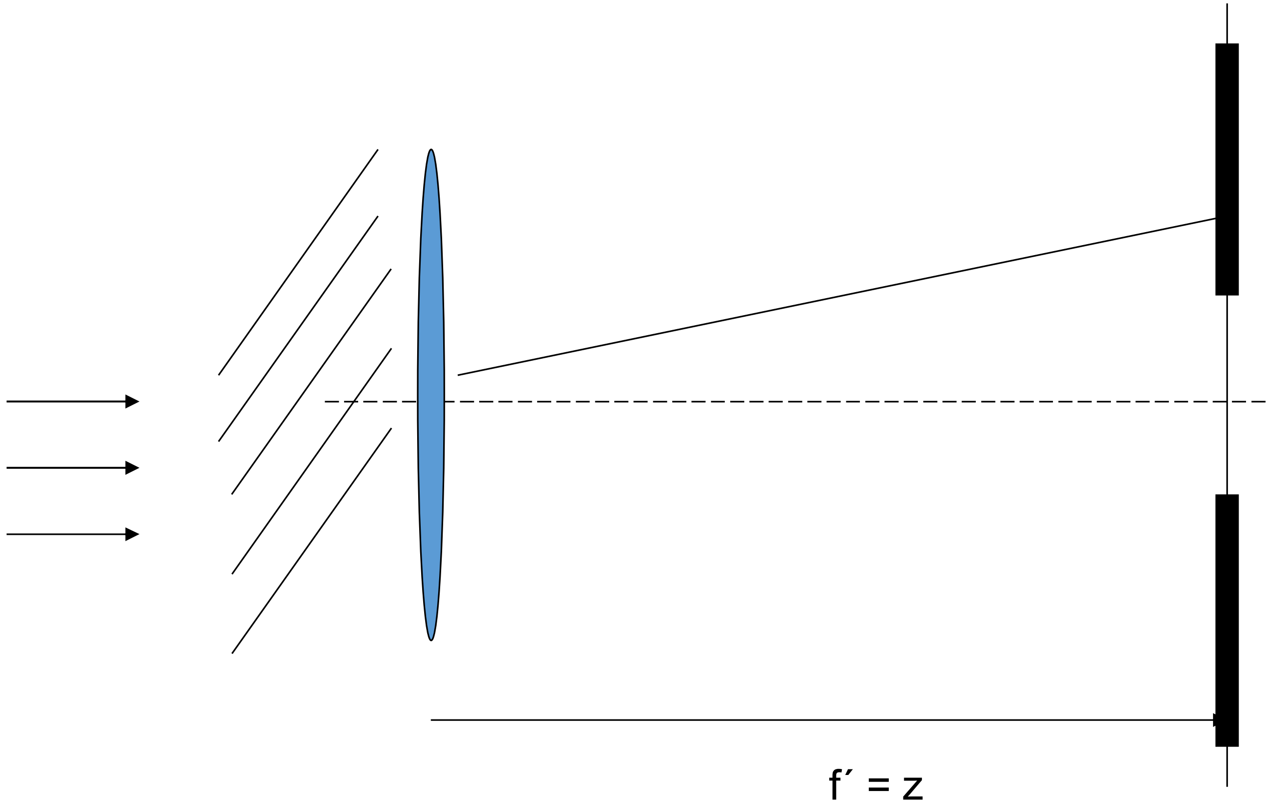
$$k_{ym} = \frac{m\,2\pi}{d}$$

$$k_{ym} = \frac{m2\pi}{d}$$

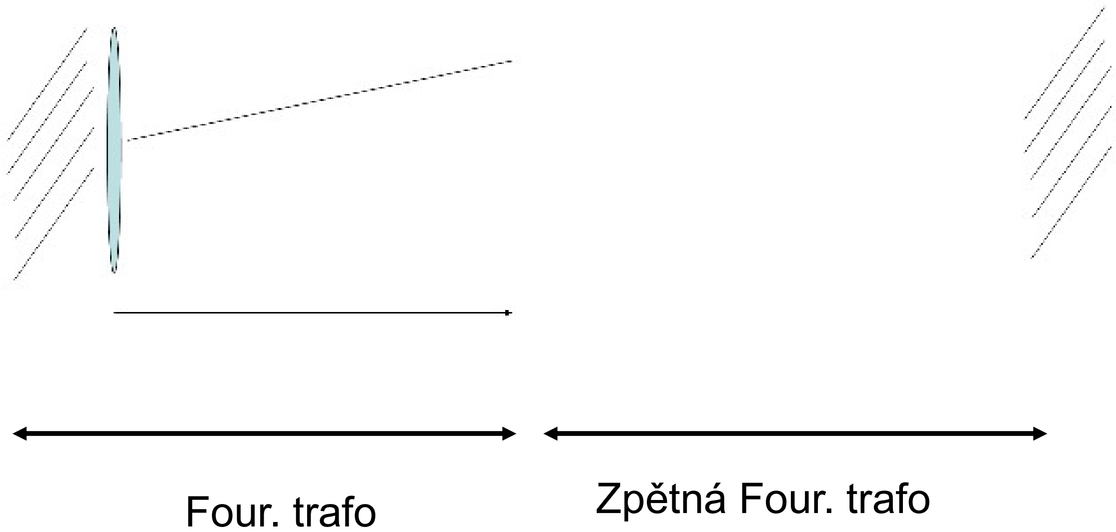
Menší mřížková konstanta *d*, větší frekvence (větší úhel)

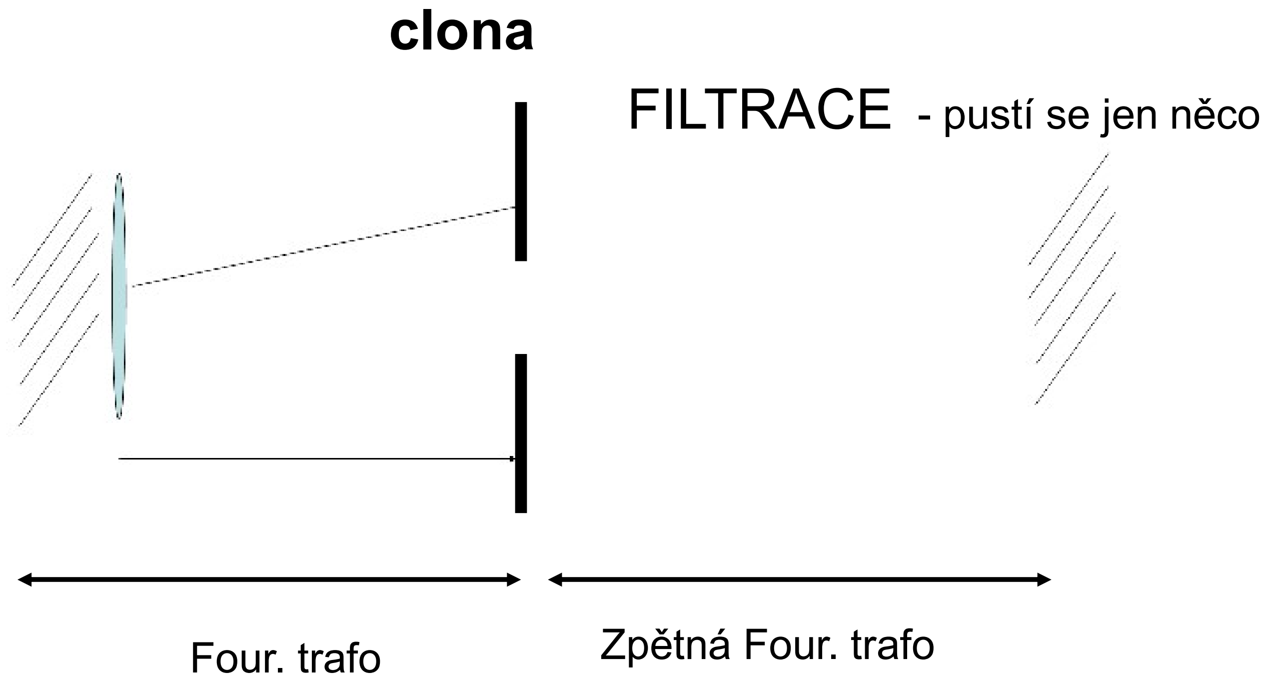


$$k_{ym} = \frac{m 2 \pi}{d}$$



$$k_{ym} = \frac{m 2\pi}{d}$$

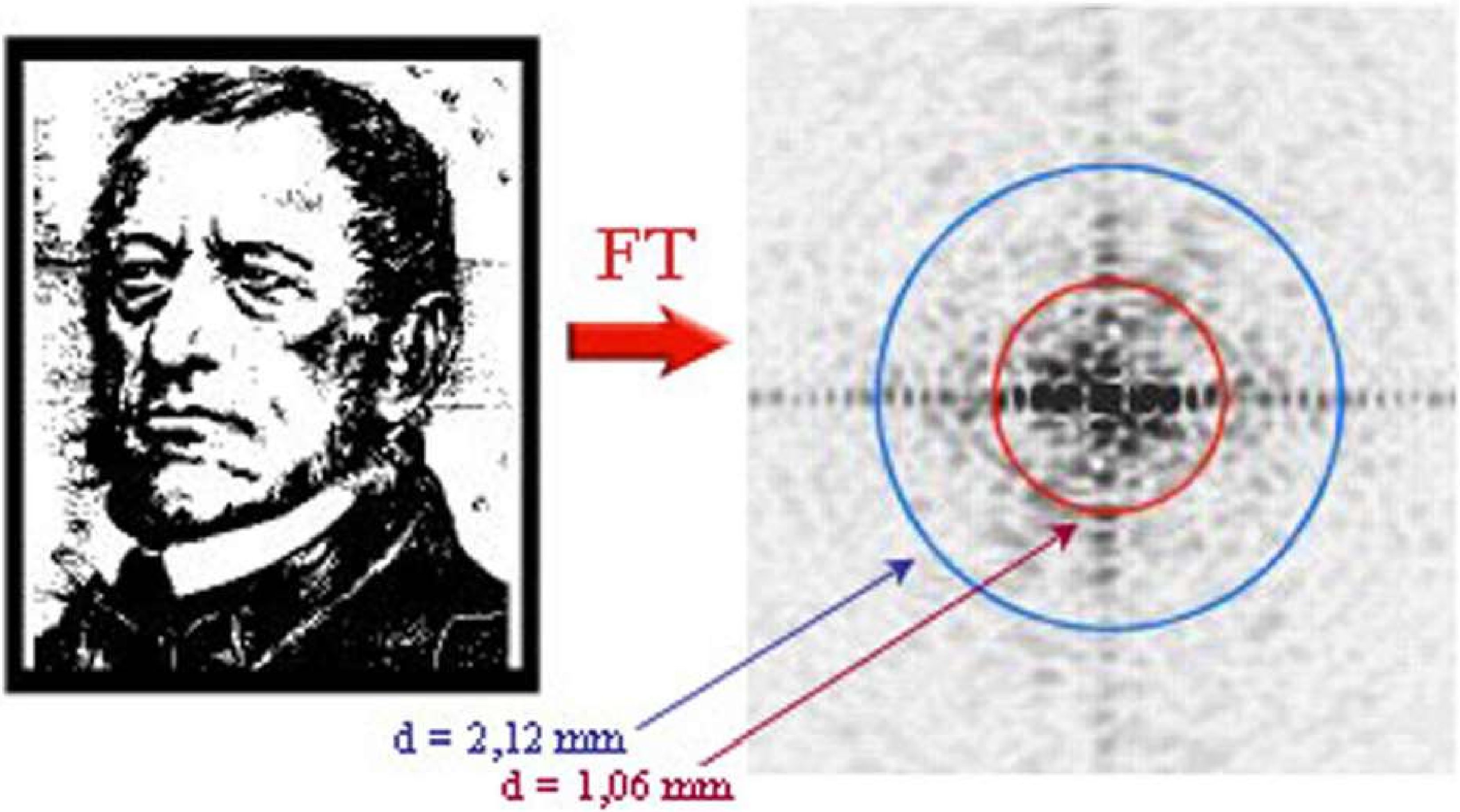

Four. trafo
Zpětná Four. trafo

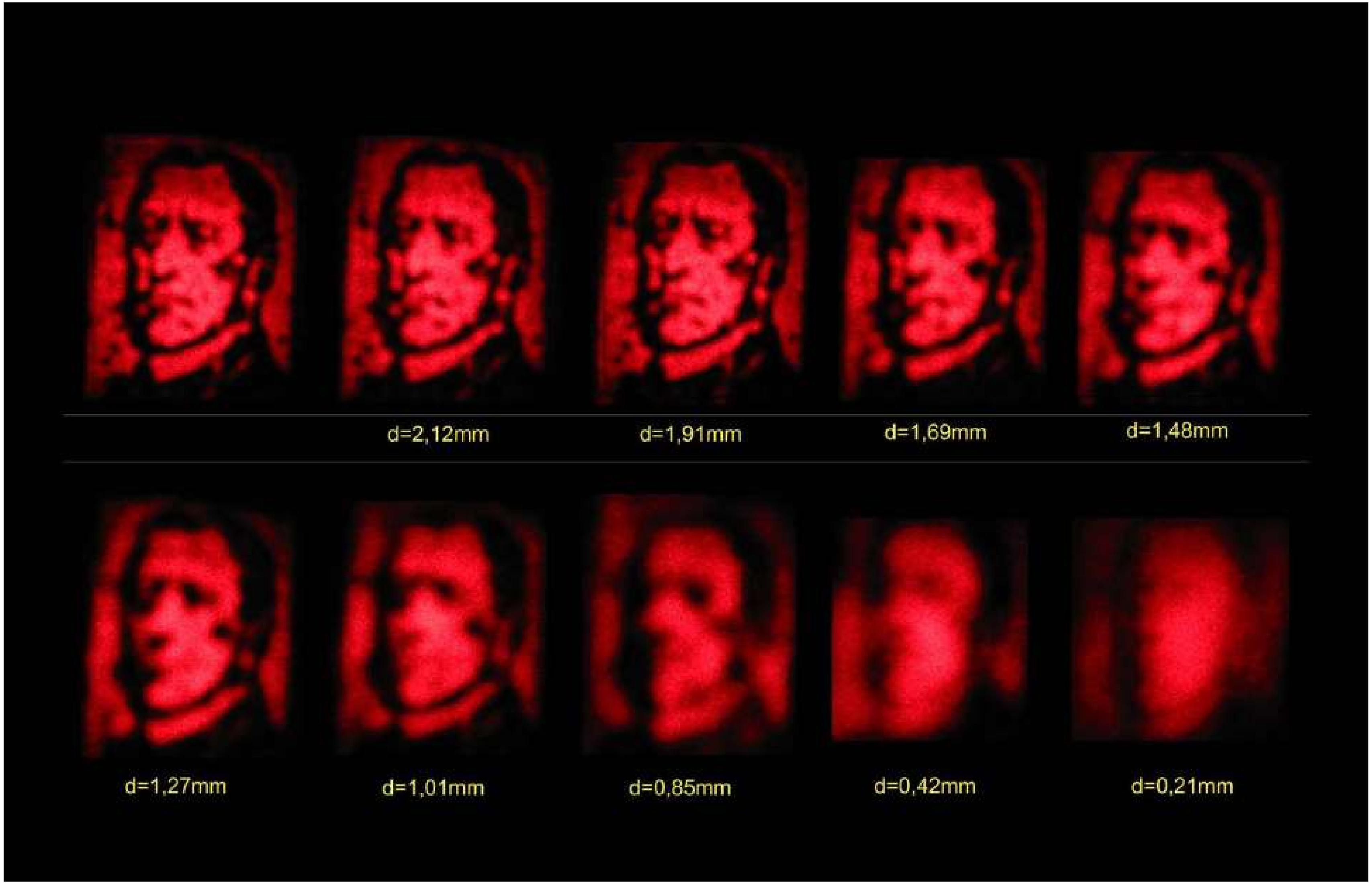

clona
FILTRACE - pustí se jen něco
Four. trafo
Zpětná Four. trafo

Optické zpracování obrazu

Experiment: J. Preclíková a K. Žídek

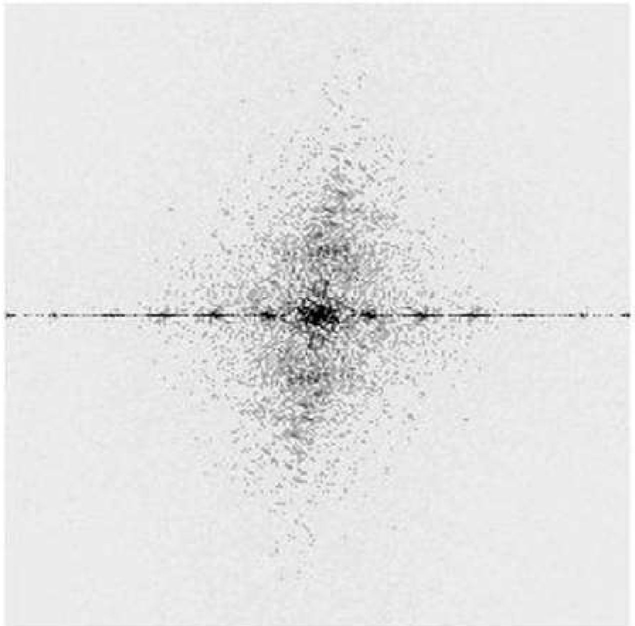
František Palacký



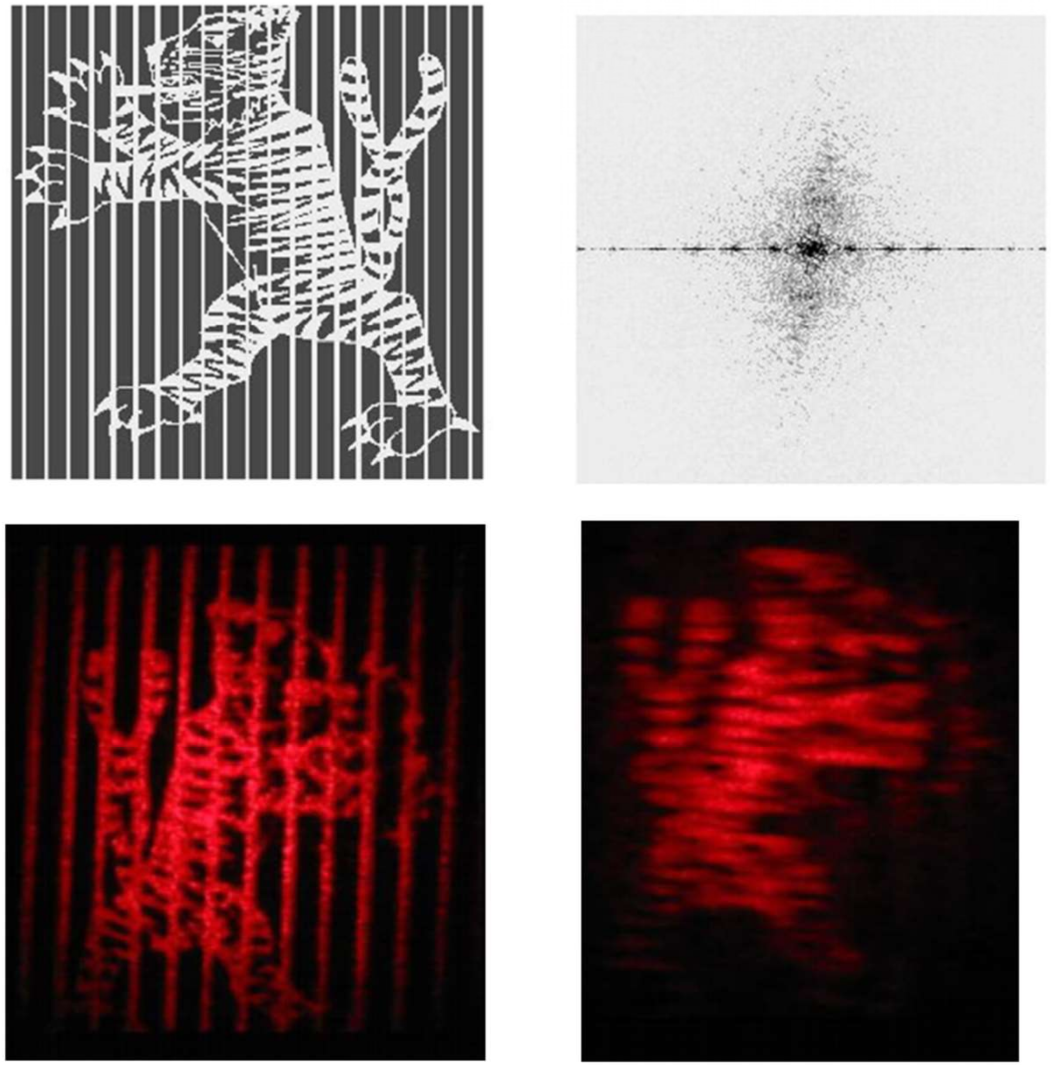


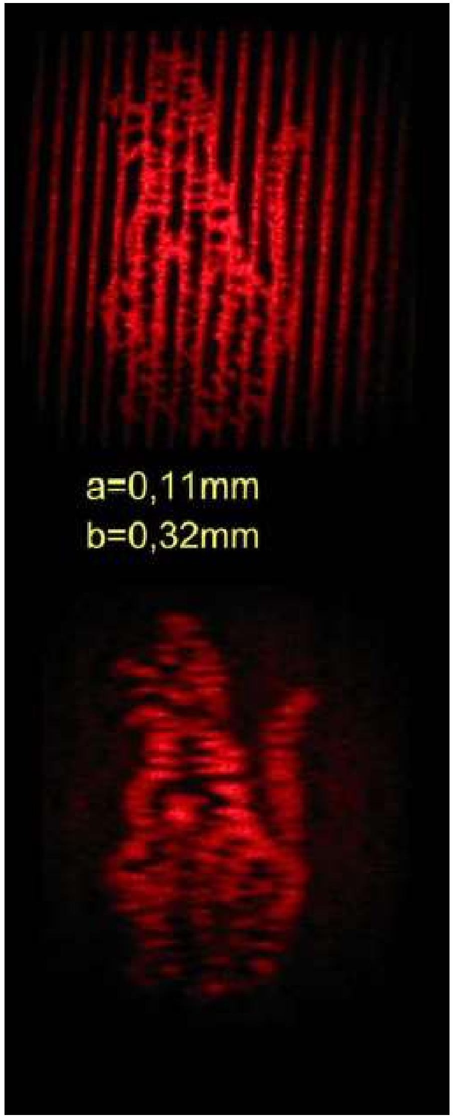
d=2,12mm
d=1,91mm
d=1,69mm
d=1,48mm
d=1,27mm
d=1,01mm
d=0,85mm
d=0,42mm
d=0,21mm

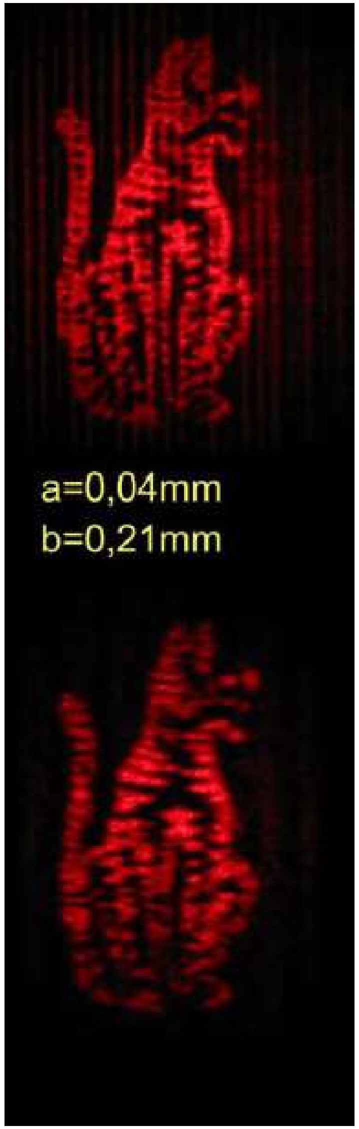
Tygr v kleci



FT




a=0,11mm
b=0,32mm


a=0,04mm
b=0,21mm