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ARBITRARY LAGRANGIAN-EULERIAN (ALE) METHODS IN PLASMA PHYSICS

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Contents

1 State of the Art

Although the field of numerical simulations is very large, our interest focuses on the simulations of the behavior of the compressible fluid, symbolized mathematically by a system of partial differential equations of hyperbolic type. Solution of such equations is represented as a density distribution of the conservative quantities in the computational domain, in the particular time of the simulation. In the case of compressible fluid dynamics, it is usual to neglect the fluid viscosity and the full set of the Navier-Stokes equations reduces to the form of system called the Euler equations. Here, the conservative quantities become the total mass, the total energy, and the components of the total momentum in each direction.

For plasma simulations, two approaches are possible – the kinetic and the fluid models. In the kinetic model, the complete Boltzmann equation is solved either by a direct or statistical method (such as Particle in Cell or Monte Carlo method), and the solution has the form of the velocity distribution of electrons and ions. Kinetic approach may provide more complete information and use less assumptions than fluid approach, but it is computationally more expensive. The fluid approach assumes additional conditions to be applicable, and its computational time is reasonable also for large-scale simulations and in higher dimensions.

There are two main approaches for numerical solving of the compressible fluid Euler equations – the Eulerian and Lagrangian methods. In the Eulerian model, the system of equations is discretized on the static (in time) computational mesh. The conservative quantities (as mass, momentum, or total energy) are transfered between the computational cells in the form of the advective flux through their edges. There exist many monographs about the Eulerian methods in computational fluid dynamics, e.g. [21, 32], and several big codes for laser plasma simulations based on the Eulerian hydrodynamics, such as `SAGE/RAGE` [11] codes.

The philosophy of the Lagrangian-type methods is completely different from the Eulerian philosophy. The computational mesh moves with the fluid, masses in all cells remain constant. There is no mass flux (and thus no advective momentum or energy flux) through the cell edges as in the Eulerian model. The Lagrangian methods are suitable for simulations, in which the computational domain size or shape dramatically changes (such as laser plasma simulations). The

mesh movement naturally treats the boundary conditions, on the other hand it can happen that the computational mesh tangles and the computation fails. The systematic approach for deriving the staggered Lagrangian schemes has been presented in [28], and used in [6] to develop the conservative Lagrangian discretization in cylindrical coordinates. One of the Lagrangian codes specialized for the laser plasma simulations is the code `ATLANT` [13].

Advantages of both approaches combines the arbitrary Lagrangian-Eulerian (ALE) method. The ALE method uses standard Lagrangian step for solving the fluid equations until the point, when the computational mesh tangles, becomes low quality, or simply until a specified number of Lagrangian steps. Then, the Eulerian part of the algorithm comes – smoothing of the computational mesh and conservative interpolation (remapping) of the state quantities to the new mesh. The Lagrangian step naturally treats domain boundaries and material interfaces, on the other hand, the Eulerian step does not allow the mesh to tangle. The ALE method was proposed in [12] and in recent years, many authors contributed to this topic. The ALE methods became a modern tool in computational fluid dynamics [9, 1]. From the top ALE codes, let us name codes `ALEGRA` [26], `CALE` [31], and `CORVUS` [2] designed for the plasma and ICF simulations. All the named ALE codes are not available for the public use.

We are interested in simulations of processes observed in experiments performed on the PALS laser facility [14, 27], especially the high velocity impact problem. This problem is hard for the Lagrangian method to be treated, shear flow of the material appears close to the disc and target interface causing tangling of the computational mesh and failure of the numerical method. On the other hand, the Eulerian method is not suitable due to the huge corona evolved through the simulation. Thus, the ALE method arises as a good compromise, which is able to handle such simulations.

2 Goals of the Dissertation Thesis

There are several main goals of the thesis. The first goal is the development of an efficient, linearity and local-bound preserving remapping algorithm for recomputation of the conservative quantities between similar computational meshes, and its generalization to the cylindrical geometry. For future use, this algorithm must be generalizable to 3D and to meshes with changing topology.
The next goal is the development of an efficient and reasonably accurate method for the simulations of laser-matter interaction and high velocity impacts, based on the fluid Arbitrary Lagrangian-Eulerian (ALE) methodology. This method have to treat severe computational mesh motion and produce results comparable with the experimental data. The method must be able to reproduce the experiments in both Cartesian and cylindrical geometries, and testing of the method on a set of experiment-inspired simulations is required.
The last main goal of the thesis is the implementation of the complete ALE method, development of the computer code, and performing simulations of laser-matter and high velocity impact problems to model selected experiments [14, 27]. Treatment of the high-velocity impact problem (not managed by purely Lagrangian methods), is required.

3 Applied Methods

Here, we describe methods used for the ALE method development and its generalization to the cylindrical coordinates. Then, we summarize performed numerical simulations of the laser-matter experiments.

3.1 ALE Algorithm

In the thesis, the hydrodynamical arbitrary Lagrangian-Eulerian method was developed. The ALE algorithm consists of the Lagrangian solver, mesh smoothing procedure, and the conservative quantity remapping method. Moreover, to allow simulations of laser plasma experiments, it incorporates routines for thermal conductivity and laser absorption. Let us overview all parts of the ALE algorithm.

3.1.1 Lagrangian Solver

The fundamental part of the ALE algorithm is the Lagrangian step solving the system of the fluid Euler equations. In our method, we use the Lagrangian solver [6, 7] employing the staggered discretization with scalar quantities (such as fluid pressure, density, or internal energy) defined in the centers of the computational mesh cells, and the vector quantities (such as positions or velocities) defined in the mesh nodes.

The Lagrangian solver is based on the computation of three types of forces in each node of the computational mesh, and movement of the nodes according to these forces. They include the zonal pressure force, the subzonal pressure force, and the artificial viscosity force. The zonal pressure force represents the total force of the fluid caused by the pressure gradient, affecting the node from all cells around it. The subzonal pressure force arises from the finer discretization (subzones), and prevents the nodes from unphysical hourglass-type motion. The viscosity force adds artificial diffusion to the solution, which makes the Lagrangian solver able to perform also simulations including shock waves or contact discontinuities. There exist many approaches, how to incorporate the artificial viscosity to the solution, we use the bulk viscosity [8], edge viscosity [8], and advanced tensor viscosity [5] in our simulations. Finally, all quantities are updated on the moved mesh.

The mentioned Lagrangian solver is conservative for all conservative quantities, and if using the second-order Runge-Kutta method (RK2) for time integration, it is second-order accurate.

3.1.2 Mesh Rezoning Algorithm

The second essential part of the ALE algorithm is the method for mesh untangling and smoothing. It regularizes the computational mesh and produces the new one, used for further calculation.
There exist many types of mesh rezoning techniques. In most of our simulations, we use the classical Winslow smoothing technique [33], based on weighted averaging of the Lagrangian mesh. This movement attempts to preserve mesh orthogonality and is used for mesh smoothing in many ALE codes [26]. Although, this method can produce meshes very different from the original Lagrangian meshes, on the other hand (unlike the plain averaging), the obtained results are reasonable and can be used for real ALE simulations. There exist some special cases, where this method is not advisable and more sophisticated method is necessary, but in general, the Winslow smoothing method can be employed to most ALE simulations.

3.1.3 Conservative Remapping

The last essential part of the ALE algorithm is the conservative interpolation of all quantities from the Lagrangian computational mesh, to the new, smoothed one. We require the remapping method to be linearity-preserving (this condition seems in practical tests to imply second order of accuracy), conservative for all conservative quantities, and local-bound preserving (the method should not create new local extrema in any of the primitive quantities). Our approach results from [23], and reduces the problem of remapping all conservative quantities to the problem of remapping of each of them by a single process, while satisfying the named properties.
The algorithm for single arbitrary conservative quantity from the Lagrangian to the smoothed mesh is based on the method introduced in [24] and extended in [20]. It consists of three parts – piecewise linear reconstruction inside old cells, numerical approximate integration of the reconstructed function, and repair. Let us review all stages briefly.

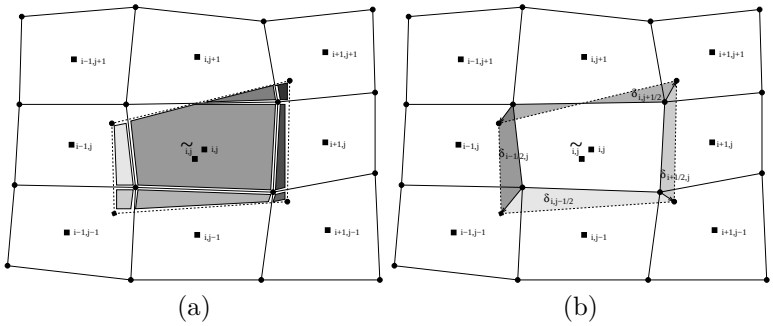


Figure 1: Exact (a) and swept (b) integration. Cell intersections and swept regions are shown in different shades of gray.

Piecewise-linear reconstruction In the first stage, the unknown function (density of the particular conservative quantity) is approximated by the piecewise linear function, which is exactly equal to its mean values in the cell centers (centroids) of the computational cells and is linear inside each old cell. The slopes in each cell are estimated by numerical minimization of the error functional defining the measure of the difference of the neighboring mean values from the (unlimited) reconstructed values in the centers of the neighboring cells. These unlimited slopes are then limited by the Barth-Jespersen limiter [3]. This limiter is constructed such that it preserves linear function, but it does not allow any overshoots in the reconstructed function.

Numerical Integration In the second stage, the reconstructed function is integrated over the new cells and new mean values are obtained. The most natural approach is the "exact integration", analytically integrating the reconstructed function over all overlapping elements of both meshes (shown in different shades of gray in Figure 1 (a)), which gives us the masses of these cells intersections. By summing the masses corresponding to some particular new cell, we get the new cell mass and density mean value. Unfortunately, due to the need to compute cells intersections, this method is computationally expensive in 2D and almost inapplicable in 3D. Thus, another method was developed – an approximate "swept integration" [24], which does not require the computation of cell intersections. This

method is based on the exchange of the cell mass through swept regions, defined by smooth movement of the original cell edges to their new positions. Four swept regions of a particular cell are shown in Figure 1 (b). The masses of the swept regions are obtained by analytical integration of the reconstructed function over swept regions. This method is efficient, global-linearity preserving, and generalizable to 3D. On the other hand, it is an approximate method. In regions of significant changes of the remapped quantities, the local extrema can be violated. So, one more step is required to enforce satisfaction of the local-bound preservation condition.

Repair Repair (conservative redistribution of conservative quantity) is the last stage of the remapping algorithm enforcing preservation of local extremes. It was introduced in [20]. Suppose, that in a particular cell its lower bound is violated, and some amount of mass is required in it to increase the value to the lower bound. For each neighboring cell, we compute the available mass, which can safely be taken from it without violating its lower bound. If the required mass is bigger than the total available mass in the neighborhood, we increase the "wrong" value to the lower bound and subtract the masses from the neighboring cells proportionally to the available masses in them. It the required mass is smaller than the total available mass, we have to extend the neighborhood and search for mass in a larger patch. We have proved that it is always possible to successfully finishes the repair process in a finite number of steps. The presented repair process is naturally conservative and does not affect the linearity preservation of the complete remapping process.
For completeness, let us note, that we have generalized the complete remapping algorithm to general 3D meshes [10], and to general 2D meshes with changing connectivity [19]. This is necessary step for the future development of 3D and 2D changing connectivity ALE codes.

3.1.4 Generalization to Cylindrical Geometry

The complete ALE algorithm was generalized to the cylindrical $r-z$ geometry. Cylindrical geometry is necessary for performing simulations of laser-plasma interactions, which are naturally cylindrical, and Cartesian simulations do not correspond to the physical reality well. In general, the generalization consists of replacing all integrations in Cartesian geometry by the cylindrical integrals.

For our discretization, there are two cylindrical Lagrangian steps available – the Area-Weighted Differencing (AWD) scheme and the Control Volume (CV) method, both reviewed in [6]. We have shown, that in our approach it is not possible to conservatively combine the AWD scheme with the rest of the ALE algorithm, and we rejected it. The CV cylindrical method is used in our cylindrical ALE code, and conservatively cooperates with the rest of the ALE algorithm.
As for the mesh smoothing stage, we considered it from a purely geometrical point of view. In the cylindrical geometry, we use the same smoothing techniques, as in the Cartesian geometry, there is almost no need to change anything.
In the remapping stage, the generalization is straightforward, the cylindrical remapping method is constructed by replacing all Cartesian integrals by cylindrical ones. The reconstruction stage does not require any changes, it does not include any integration. The same situation arises in the repair stage, which only moves masses between cells, no integration appears in it. Different situation is during the swept integration stage, where the swept masses are computed by integration of the corresponding reconstructed function. Here, the cylindrical integrals must appear.
The complete cylindrical ALE method provides reasonable solutions of cylindrical fluid tests, and have the same properties, as the Cartesian one.

3.2 Physical Aspects of ALE Simulations

Let us briefly review the physical aspects of the ALE simulations, such as the plasma equation of state, incorporation of thermal conductivity and laser absorption to the ALE method.

3.2.1 Equation of State

Equation of state is a very important part of all fluid and plasma simulation codes. It is necessary for the computation of the actual fluid pressure and temperature from the actual fluid density and internal energy, and vice versa.
For simple fluid tests, the classical ideal gas equation of state (IG EOS) can be applied. For the simulations of laser-matter interactions and high velocity impact problems, the IG EOS does not provide realistic formulas for the relations among the state quantities. Thus, we

use the Quotidian equation of state (QEOS) introduced in [25]. This equation of state is valid in a broad range of laser plasma parameters, and handles both the pure elements and compound materials.

3.2.2 Thermal Conductivity

The importance of heat conductivity is different for each particular problem simulated. For some problems, such as the impact of the high-speed flyer to the massive target, the hydrodynamical effects are much stronger than the heat conductivity term, and the solutions of the same problem with and without the heat conductivity are close to each other. On the other hand, when the interaction of the laser beam with the target material is simulated, the solution without the thermal conductivity is clearly unrealistic.
Thermal conductivity effects are mathematically described by the parabolic part in the energy equation of the system of fluid Euler equations. This particular equation is solved separately by splitting from the hyperbolic part of the system. For the evaluation of the heat conductivity coefficient, the classical Spitzer-Harm [30] formula corrected by the electron-electron collision term is used.
Due to the non-linear dependence of the heat conductivity coefficient on temperature, one can expect non-linear effects such as heat waves. The numerical method for solving the parabolic equation have to be able to deal with them. The discretization of the heat conductivity equation is treated by the mimetic method [29] using support operators [28]. The fast converging conjugate gradient method can be used to solve the implicit scheme, which works well also on bad quality meshes appearing in Lagrangian simulations, and also for discontinuous diffusion coefficients.

3.2.3 Laser Absorption

For employing the interaction of the laser beam with the material, we use the simplest model available – the laser beam penetrates the material till the critical density, where it absorbs. The laser beam absorption is represented by the divergence of the laser beam intensity on the right hand side of the energy equation. By construction, this divergence is non-zero only in cells intersected by the critical density isoline. In all other cells (both sub and supercritical), the intensity divergence is neglectable and no absorption appears.

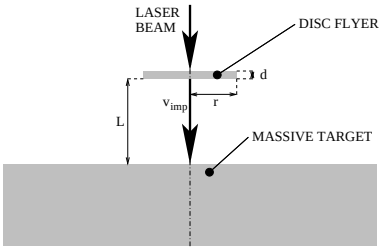


Figure 2: Setup of double-target experiment.

3.3 Numerical Simulations

Let us briefly describe the numerical simulations of real experiments performed on the PALS laser facility, which we modeled using our ALE code. After the simulations of massive target irradiation problem discussed in [4], we focus to the double-target experiment simulations from [14, 27]. Initial conditions of the double-target experiment are shown in Figure 2.

3.3.1 Massive Target Irradiation by Laser Beam

The first class of simulations is inspired by experiments [4] demonstrating the irradiation of the Aluminum massive target by an intense laser beam of the 400 ps pulse. Several experiments were performed, with different laser parameters – laser beam energy either 100 J or 600 J for first of third harmonic, and with the radius of laser spot on target varying between 35 μm and 600 μm.
The complete ALE algorithm was applied to simulate these problems. We presented final computational mesh and distributions of plasma density, temperature, and internal energy increase. The huge low-density corona moving out of the target is enormous, when compared with the original computational domain. Shock wave in the target region spreads and causes heating, melting, and evaporation of the target material. We have compared the crater (interface between gas and liquid phases) size estimated from material temperature and internal energy increase with the Cartesian simulations, and with the experimental data. The cylindrical simulations reasonably correspond to the experimental data.

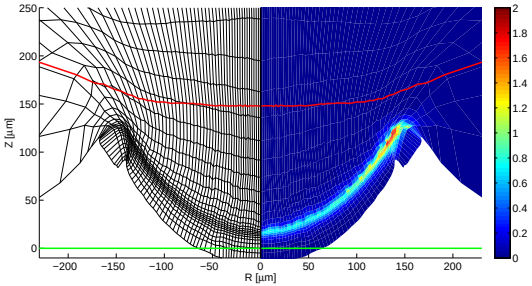


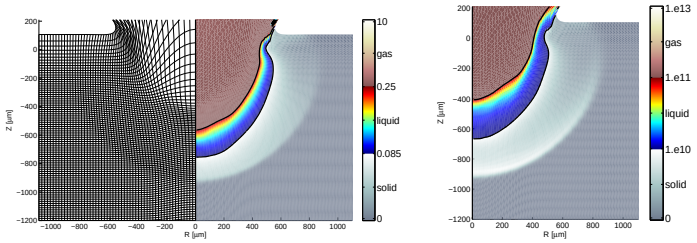
Figure 3: Density (in g/cm^3) and computational mesh in the massive part of the accelerated 11 μm disc by 240 J laser beam on the third harmonic at time of impact (1.3 ns after the start of the simulation).

3.3.2 Ablative Flyer Acceleration by Laser Beam

The next class of simulations is the acceleration of a thin (6 μm or 11 μm) Aluminum disc by an intense laser beam with energy 120 J, 130 J, 240 J, or 390 J in 400 ps laser pulse in first or third harmonic, with the 125 μm diameter of the laser spot on target [14, 27]. The setup of the experiment is presented in Figure 2.
The laser pulse evaporates the surface of the disc, and the rest is ablatively accelerated to a high velocity (up to 200 km/s). Zoom to the massive part of the particular accelerated disc is shown in Figure 3. The experimental velocities are compared to the average disc velocities obtained from the preliminary 1D Lagrangian simulations [16] and from complete 2D ALE simulations. The 2D ALE simulations correspond to the experimental data much better than the 1D ones, and reasonably approximate experimental disc velocities. Several 2D acceleration simulations have been already presented in [22], [15].

3.3.3 High Velocity Impact Simulations

The last family of problems which we simulate by our 2D ALE code is the impact of the accelerated disc flyer to the massive Aluminum target [14, 27]. The previous massive and flyer target irradiation problems are possible to be simulated by a purely Lagrangian method (even though with problems), the high velocity impact problem requires the complete ALE methodology. We have demonstrated the



(a) mesh and temperature (b) internal energy increase

Figure 4: Computational mesh (only every second edge in each logical direction is visible) and temperature (in eV) (a) and specific internal energy increase (in erg/g) (b) of 6 μm thick disc irradiated by 130 J laser beam on third harmonic at time $t = 80$ ns after the impact. Solid, liquid and gas phases are separated by isolines in temperature and energy increase of melting and evaporation of Aluminum, different material phases are shown in different colormaps.

failure of the purely Lagrangian method in the early stages of the simulation.

The quantities (density, temperature, and velocity fields) from the disc flyer simulation (Figure 3) are interpolated to the initial mesh of the impact simulation. After the impact, a shock wave is created, which (as in the massive target irradiation simulations) spreads inside the target and causes its heating (see Figure 4). The crater is defined by the gas/liquid interface again, and its size is estimated from both temperature and internal energy increase distributions. We have compared the sizes of craters obtained by the described simulations and craters obtained by simulations started from the initial data coming from 1D Lagrangian simulations of the flyer acceleration (presented in [17, 18]). We have shown, that the simulations started from the interpolated initial data provide better results, and that the simulated craters are comparable with the experimental data.

3.3.4 Energy Balance of the Simulation

To show the consistency of our laser-flyer-target simulations, we summarized the kinetic, internal, and total energies in several stages of the simulations. We have shown, that most of the kinetic energy in

the accelerated flyer is deposited in the high density region of the impacting disc. This energy is then transformed to the energy of expanding corona and to the energy of the heating target. The behavior corresponds reasonably to the expected process, and shows the ability of the code to perform relevant laser plasma simulations.

4 Outputs and Contributions

The main contributions of the thesis to the problematics of conservative interpolations, ALE methods, and computational laser plasma hydrodynamics, are listed below:

- Introducing the swept region remapping method in
  - 2D logically quadrilateral meshes in Cartesian and cylindrical geometries.
  - 3D general meshes.
  - 2D general meshes with changing connectivity.
- Introducing the complete remapping algorithm (for all state quantities) in cylindrical geometry.
- Development of the 2D ALE code on logically-orthogonal computational meshes working in both Cartesian and cylindrical geometries, applicable to the fluid and laser plasma simulations.
- Simulations of the small disc flyer ablative acceleration by an intense laser beam, and its impact to the massive target.

The conservative interpolation (remapping) method is based on piecewise-linear reconstruction, approximate swept region integration, and repair stage enforcing local-bound preservation. We have also introduced the generalization of the remapping algorithm to general 3D meshes, and 2D meshes with changing connectivity. To recompute all state quantities in both geometries, we have generalized the algorithm from [23] to the cylindrical geometry. The presented complete ALE method (in both geometries) was implemented into a computer code in Fortran. Its memory and computational time demands are reasonable on current machines. Finally, we performed several simulations of laser-target and laser-flyer-target experiments, and present the obtained results. The simulations follow the real experiments performed on PALS laser system [4, 14, 27], the simulated and experimental craters are comparable.
The given points are new, and can be useful for both the ALE and laser plasma societies.

5 Conclusion

In the thesis "Arbitrary Lagrangian-Eulerian (ALE) Methods in Plasma Physics", the author has described the complete arbitrary Lagrangian-Eulerian (ALE) method for fluid dynamics and laser plasma simulations on 2D logically orthogonal computational meshes, in Cartesian and cylindrical geometries. Its properties have been demonstrated for selected problems of laser-plasma interactions.

Our ALE method uses the staggered Lagrangian step [6], several mesh smoothing techniques, and the conservative remapping process [24]. We have focused in details on the remapping algorithm and introduced several its improvements [20]. The complete ALE algorithm was generalized into the cylindrical geometry, allowing to perform simulations of naturally cylindrical laser-plasma processes.

The ALE method has been implemented, and several techniques for treating the laser plasma behavior – QEOS equation of state [25], thermal conductivity, simple laser absorption model, or dynamically changing boundary conditions – have been added to it. These techniques allow the presented method to be used for advanced laser-plasma simulations. We have demonstrated its properties on a set of laser-flyer-target simulations based on real experiments, and compared the simulations with the experimental results. Their correspondence and the energy balance analysis show the ability of the code to provide realistic simulations for problems, where neither the Eulerian nor Lagrangian approaches are suitable.

The methods described in this thesis give powerful tools to those who need to perform laser plasma simulations, for which the classically used Eulerian or Lagrangian methods have troubles. It can be used for such problems, as high velocity impact simulations, and it has been shown to produce realistic results. The field of laser plasma physics is a dynamically evolving topic influencing many aspects of the human being, and the demands of implementing new methods into our code can be expected in the future.

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List of Related Important Publications

For completeness, we add the list of selected important journal and conference proceedings articles of the thesis author, connected to the thesis topic. Besides the presented list of publications, the author published several technical reports, short reports in university and group collections, and abstracts in the conference books of abstracts. The author also presented the topic on many local meetings and international conferences, and keeps in touch with the scientific community. The complete list of publications of the author is enclosed.

Journal articles:

1. R. Garimella, M. Kuchařík, and M. Shashkov: An Efficient Linearity and Bound Preserving Conservative Interpolation (Remapping) on Polyhedral Meshes, *Computers and Fluids*, 2006. In press.

2. M. Kuchařík, R. Liska, J. Limpouch, and P. Váchal: ALE Simulations of High-Velocity Impact Problem, *Czechoslovak Journal of Physics*, Vol. 54, Suppl. C, pp. 391–396, 2004.

3. M. Kuchařík, M. Shashkov, and B. Wendroff: An efficient linearity-and-bound-preserving remapping method, *Journal of Computational Physics*, Vol. 188, Nr. 2, pp. 462–471, 2003.

Articles in Conference Proceedings:

1. R. Liska, and M. Kuchařík: Arbitrary Lagrangian Eulerian method for compressible plasma simulations, *Proceedings of EQUADIFF 11, International conference on differential equations*, Comenius University, 2005. 10 pages. Submitted.

2. M. Kuchařík, J. Limpouch, and R. Liska: Laser Plasma Simulations by Arbitrary Lagrangian Eulerian Method, *Proceedings of IFSA 2005, Fourth International Conference on Inertial Fusion Sciences and Applications*, 2005. 3 pages. Submitted.

3. M. Kuchařík: Conservative Interpolations in ALE Codes, *Proceedings of Workshop of Applied Mathematics 2005*, Czech Technical University in Prague, 2005. 9 pages. Accepted.

4. M. Kuchařík, R. Liska, and M. Shashkov: Conservative Remapping and ALE Methods for Plasma Physics, *Proceedings of HYP 2004, Hyperbolic Problems: Theory, Numerics and Applications*, volume 2, editors F. Asakura, S. Kawashima, A Matsumura, S. Nishibata, and K. Nishihara. Yokohama Publishers, 2006. 8 pages. ISBN 4-946552-22-7.

5. M. Kuchařík, J. Limpouch, R. Liska, and P. Havlík: ALE Simulations of Laser Interactions with Flyer Targets, *Proceedings of XXVIII ECLIM, 28th European Conference on Laser Interaction with Matter*, pp. 470–474, 2004.

6. M. Kuchařík, and R. Liska: Arbitrary Lagrangian-Eulerian (ALE) Code for Plasma Simulations, *Proceedings of Czech-Japanese Seminar in Applied Mathematics*, Czech Technical University in Prague, editors M. Beneš, J. Mikiška, and T. Oberhuber, pp. 96–105, 2005. ISBN 80-01-03181-0.

7. R. Garimella, M. Kuchařík, and M. Shashkov: Efficient Algorithm for Local-Bound-Preserving Remapping in ALE Methods, *Proceedings of ENUMATH 2003, Numerical Mathematics and Advanced Applications*, editors M. Feistauer, V. Dolejší, P. Knobloch, and K. Najzar. Springer-Verlag Berlin Heidelberg New York, pp. 358–367, 2004. ISBN: 3-540-21460-7.

8. M. Kuchařík, M. Shashkov, and B. Wendroff: Efficient Local Bound-Preserving Conservative Interpolation, *Proceedings of Seventh U.S. National Congress on Computational Mechanics*, Omnipress, USA, pp. 166, 2003. ISBN: 0-9743254-0-6.

Resume (EN)

The complete Arbitrary Lagrangian-Eulerian (ALE) method, applicable to the fluid and laser plasma hydrodynamics, both in Cartesian and cylindrical geometries is presented. All parts of the ALE algorithm, i.e. Lagrangian solver, mesh smoothing, and conservative quantity remapping, in both geometries are fully described. The issues related to the laser plasma physics as plasma equation of state, thermal conductivity, interaction with laser beam, and sophisticated treatment of the boundary conditions necessary for the realistic laser plasma simulations are described. The complete developed ALE code is tested on a series of typical fluid problems to show its properties for the well known solutions. Finally, three sets of laser plasma simulations inspired by the real experiments are performed – the interaction of a laser beam with a massive target, ablative acceleration of small Aluminum disc flyer irradiated by a laser beam, and the high velocity impact of such accelerated disc onto a massive Aluminum target. The standard Lagrangian simulation of the last, high velocity impact problem fails, and the complete ALE methodology is required for this problem. Simulations of all types of problems show reasonable agreement with the experimental results.

Resumé (CZ)

Disertační práce "Arbitrary Lagrangian-Eulerian (ALE) Methods in Plasma Physics" (Lagrangeovsko-Eulerovské metody ve fyzice plazmatu) popisuje kompletní ALE Lagrangeovsko-Eulerovský algoritmus aplikovatelný na hydrodynamiku tekutin a plazmatu, a to v kartézské a cylindrické geometrii. Práce popisuje jednotlivé části ALE algoritmu v obou souřadnicových systémech – Lagrangeovský řešič, metody pro vyhlazování výpočetních sítí a pro interpolaci (remapování) zachovávajících se veličin mezi nimi. Dále jsou popsány procesy spojené se simulacemi fyziky laserového plazmatu, jako je stavová rovnice plazmatu, mechanizmus tepelné vodivosti, iterakce s laserovým paprskem a důmyslné ošetření okrajových podmínek, nutné pro realistické simulace jevů v plazmatu. Abychom ukázali vlastnosti presentované metody, je vyvinutý ALE kód je otestován na sadě typických problémů dynamiky tekutin se známým řešením. V poslední části simulujeme tři sady problémů inspirovaných skutečnými experimenty – interakci laserového paprsku s masivním terčem, ablativní urychlení malého hliníkového disku po dopadu laserového záření a vysokorychlostní dopad takto urychleného disku na masivní hliníkový terč. Pro poslední zmiňovaný problém (vysokorychlostní dopad) selhávají standardní Lagrangeovské metody a je nutná kompletní procedura ALE. Simulace všech typů problémů přiměřeně odpovídají experimentálním výsledkům.